The greatest mathematical achievement of the 20th century

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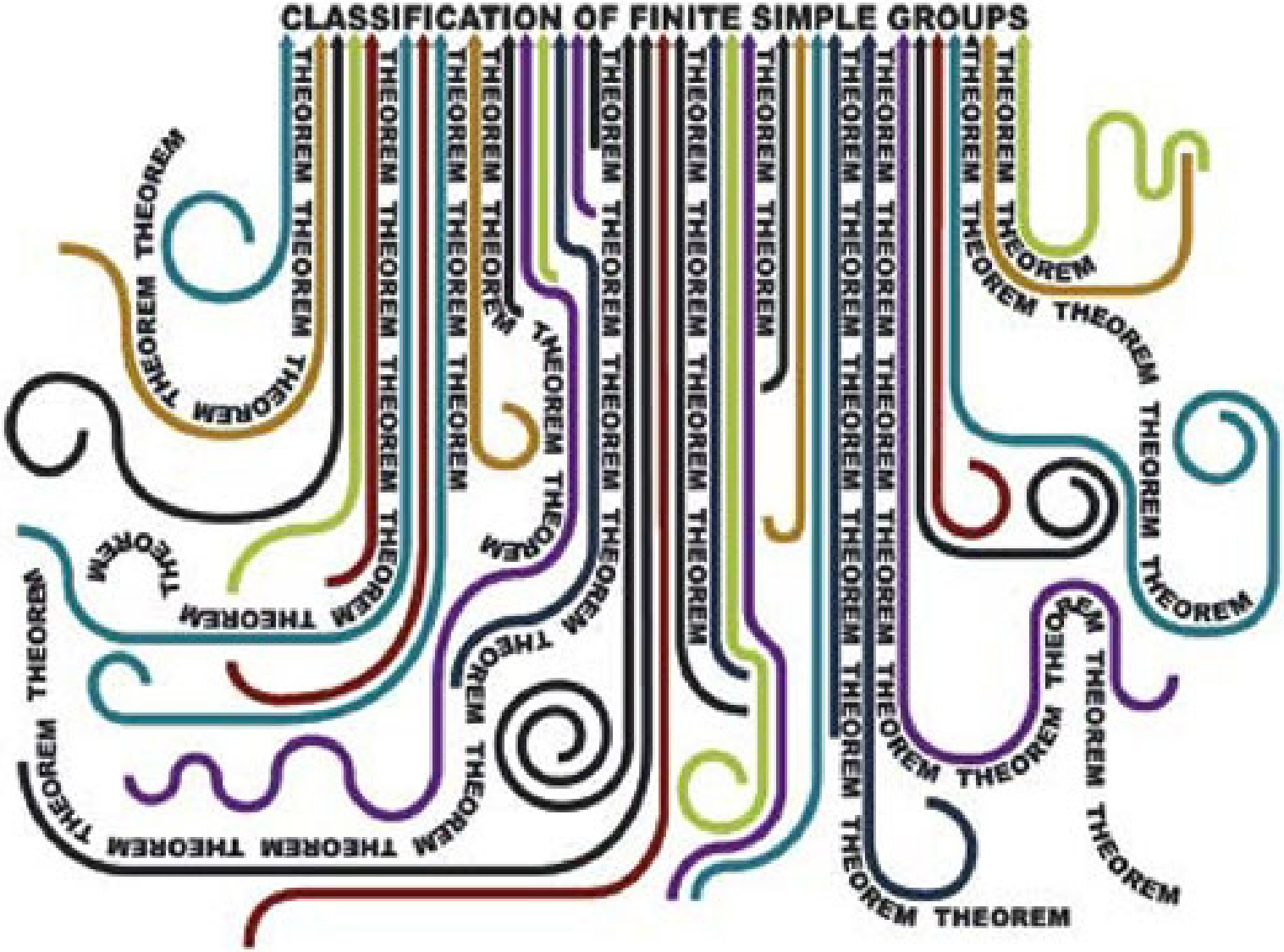
great (grāt), *adj*. 1. of an extent, amount, or intensity considerably above the normal or average; very large and imposing.



http://plus.maths.org/content/enormous-theorem-classification-finite-simple-groups

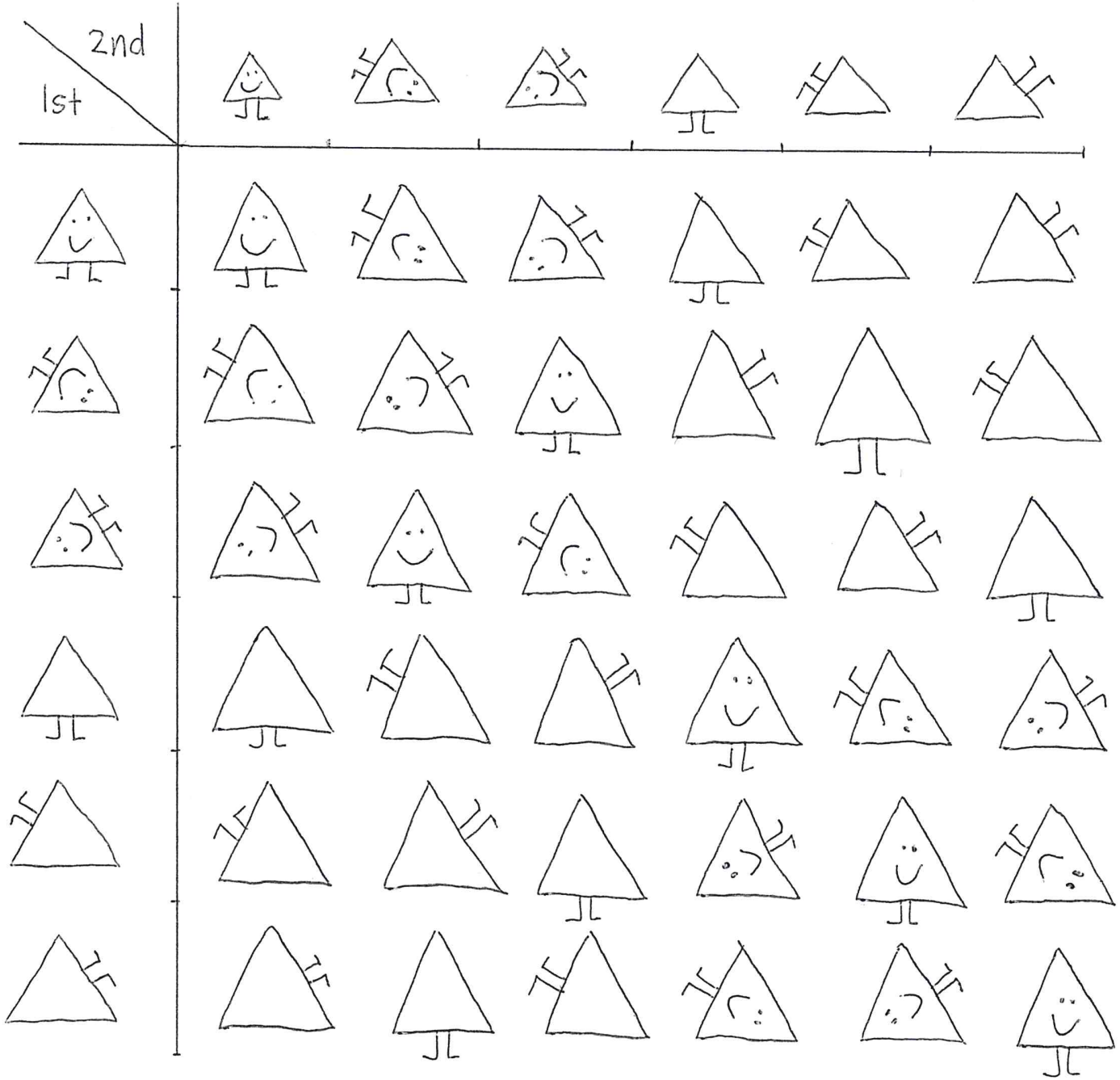
First-generation proof of CFSG:

- 10,000–15,000 journal pages
- spread across some 500 separate articles, written by more than 100 mathematicians from around the world
- most work done between 1950 and early 1980s

Second-generation proof of CFSG (currently underway):

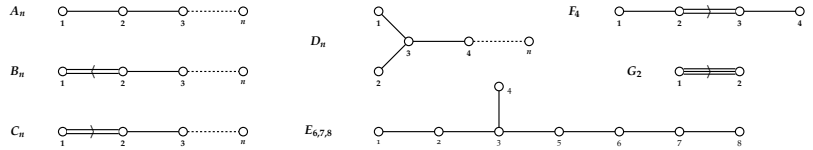
- shorter, more direct proof of 3,000–4,000 pages
- volume 1 published in 1994
- volume 6 published in 2005 (most recent volume)
- 12 volumes anticipated

What is a (finite simple) group?


2nd
1st

The Periodic Table Of Finite Simple Groups

Dynkin Diagrams of Simple Lie Algebras



$0, C_1, \mathbb{Z}_1$ 1 1																	C_2 2
$A_1(4), A_1(5)$ A_5 60	$A_2(2)$ $A_1(7)$ 168											${}^2A_3(4)$ $B_2(3)$ 25 920	$C_3(3)$ 4 585 351 680	$D_4(2)$ 174 182 400	${}^2D_4(2^2)$ 197 406 720	$G_2(2)'$ ${}^2A_2(9)$ 6 048	C_3 3
$A_1(9), B_2(2)'$ A_6 360	${}^2G_2(3)'$ $A_1(8)$ 504											$B_2(4)$ 979 200	$C_3(5)$ 228 501 000 000 000	$D_4(3)$ 4 952 179 814 400	${}^2D_4(3^2)$ 10 151 968 619 520	${}^2A_2(16)$ 62 400	C_5 5
A_7 2 520	$A_1(11)$ 660	$E_6(2)$ 214 841 575 522 005 575 270 400	$E_7(2)$ 7 997 476 042 075 799 759 100 487 262 680 802 918 400	$E_8(2)$ 337 804 750 140 604 806 261 883 190 414 265 205 279 993 442 667 601 376 362 909 909 263 800 000	$F_4(2)$ 3 311 126 603 366 400	$G_2(3)$ 4 245 696	${}^3D_4(2^3)$ 211 341 312	${}^2E_6(2^2)$ 76 532 479 683 774 853 939 200	${}^2B_2(2^3)$ 29 120	Tits* ${}^2F_4(2)'$ 17 971 200	${}^2G_2(3^3)$ 10 073 444 472	$B_3(2)$ 1 451 520	$C_4(3)$ 65 784 756 654 489 600	$D_5(2)$ 23 499 295 948 800	${}^2D_5(2^2)$ 25 015 379 558 400	${}^2A_2(25)$ 126 000	C_7 7
$A_3(2)$ A_8 20 160	$A_1(13)$ 1 092	$E_6(3)$ 7 257 703 347 541 463 210 028 258 395 254 643 200	$E_7(3)$ 1 271 375 236 818 136 742 240 479 751 139 022 444 554 379 203 770 766 254 617 395 200	$E_8(3)$ [illegible]	$F_4(3)$ 5 734 420 792 816 671 844 761 600	$G_2(4)$ 251 596 800	${}^3D_4(3^3)$ 20 560 831 566 912	${}^2E_6(3^2)$ 144 636 855 916 969 695 633 965 120 680 532 377 600	${}^2B_2(2^5)$ 32 537 600	${}^2F_4(2^3)$ 264 905 352 699 586 176 614 400	${}^2G_2(3^5)$ 49 825 657 439 340 552	$B_2(5)$ 4 680 000	$C_3(7)$ 273 457 218 604 953 600	$D_4(5)$ 8 911 539 000 000 000 000	${}^2D_4(4^2)$ 67 536 471 195 648 000	${}^2A_3(9)$ 3 265 920	C_{11} 11
A_9 181 440	$A_1(17)$ 2 448	$E_6(4)$ 85 528 710 781 342 640 103 833 639 055 142 765 466 746 880 000	$E_7(4)$ 131 151 438 134 960 881 379 897 220 477 804 941 284 464 149 027 318 106 347 251 710 243 504 508 800 000 000	$E_8(4)$ [illegible]	$F_4(4)$ 19 009 825 523 840 945 451 297 669 120 000	$G_2(5)$ 5 859 000 000	${}^3D_4(4^3)$ 67 802 350 642 790 400	${}^2E_6(4^2)$ 85 696 576 147 617 709 485 896 772 387 584 993 695 360 000 000	${}^2B_2(2^7)$ 34 093 383 680	${}^2F_4(2^5)$ 1 318 633 155 799 591 447 702 161 609 782 722 560 000	${}^2G_2(3^7)$ 239 189 910 264 352 349 332 632	$B_2(7)$ 138 297 600	$C_3(9)$ 54 025 731 402 499 584 000	$D_5(3)$ 1 289 512 799 941 305 139 200	${}^2D_4(5^2)$ 17 880 203 250 000 000 000	${}^2A_2(64)$ 5 515 776	C_{13} 13
A_n $\frac{n!}{2}$	$PSL_{n+3}(q), L_{n+1}(q)$ $A_n(q)$ $\frac{q^{n(n+3)/2}}{(n+3,q-1)}\prod_{i=1}^{n}(q^{i+1}-1)$	$E_6(q)$ $\frac{q^{36}(q^{12}-1)(q^8-1)(q^6-1)}{(3,q-1)}$	$E_7(q)$ $\frac{q^{63}}{(2,q-1)}\prod_{i=2}^{6}(q^{2i}-1)$	$E_8(q)$ $\frac{q^{120}(q^{30}-1)(q^{24}-1)}{(q^{12}-1)(q^{10}-1)(q^8-1)(q^2-1)}$	$F_4(q)$ $q^{24}(q^{12}-1)(q^8-1)(q^6-1)$	$G_2(q)$ $q^6(q^6-1)(q^2-1)$	${}^3D_4(q^3)$ $q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$	${}^2E_6(q^2)$ $q^{36}(q^{12}-1)(q^8+1)(q^6-1)(q^2-1)(q^5+1)(q^2-1)$	${}^2B_2(2^{2n+1})$ $q^2(q^2+1)(q-1)$	${}^2F_4(2^{2n+1})$ $q^{12}(q^6+1)(q^4-1)(q^2+1)(q-1)$	${}^2G_2(3^{2n+1})$ $q^3(q^3+1)(q-1)$	$O_{2n+1}(q), \Omega_{2n+3}(q)$ $B_n(q)$ $\frac{q^{n^2}}{(2,q-1)}\prod_{i=1}^{n}(q^{2i}-1)$	$PSp_{2n}(q)$ $C_n(q)$ $\frac{q^{n^2}}{(2,q-1)}\prod_{i=1}^{n}(q^{2i}-1)$	$O_{2n}^+(q)$ $D_n(q)$ $\frac{q^{n(n-1)}(q^n-1)}{(4,q^n-1)}\prod_{i=1}^{n-1}(q^{2i}-1)$	$O_{2n}^-(q)$ ${}^2D_n(q^2)$ $\frac{q^{n(n-1)}(q^n+1)}{(4,q^n+1)}\prod_{i=1}^{n-1}(q^{2i}-1)$	$PSU_{n+1}(q)$ ${}^2A_n(q^2)$ $\frac{q^{n(n+1)/2}}{(n+1,q+1)}\prod_{i=2}^{n+1}(q^i-(-1)^i)$	$\mathbb{Z}_p$ C_p p

- Alternating Groups
- Classical Chevalley Groups
- Chevalley Groups
- Classical Steinberg Groups
- Steinberg Groups
- Suzuki Groups
- Ree Groups and Tits Group*
- Sporadic Groups
- Cyclic Groups

*The Tits group ${}^2F_4(2)'$ is not a group of Lie type, but is the (index 2) commutator subgroup of ${}^2F_4(2)$. It is usually given honorary Lie type status.

The groups starting on the second row are the classical groups. The sporadic suzuki group is unrelated to the families of Suzuki groups.

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Alternates‡
Symbol
Order‡

†For sporadic groups and families, alternate names in the upper left are other names by which they may be known. For specific non-sporadic groups these are used to indicate isomorphisms. All such isomorphisms appear on the table except the family $B_n(2^m) \cong C_n(2^m)$.

‡Finite simple groups are determined by their order with the following exceptions:
$B_n(q)$ and $C_n(q)$ for q odd, $n > 2$;
$A_8 \cong A_3(2)$ and $A_2(4)$ of order **20160**.

M_{11} 7 920	M_{12} 95 040	M_{22} 443 520	M_{23} 10 200 960	M_{24} 244 823 040	$J(1), J(11)$ J_1 175 560	HJ J_2 604 800	HJM J_3 50 232 960	J_4 86 775 571 046 077 562 880	HS 44 352 000	McL 898 128 000	F_7, HHM, HTH He 4 030 387 200	Ru 145 926 144 000	
Sz Suz 448 345 497 600	$O'NS, O\text{–}S$ $O'N$ 460 815 505 920	$\cdot 3$ Co_3 495 766 656 000	$\cdot 2$ Co_2 42 305 421 312 000	$\cdot 1$ Co_1 4 157 776 806 543 360 000	F_3, D HN 273 030 912 000 000	LyS Ly 51 765 179 004 000 000	F_3, E Th 90 745 943 887 872 000	$M(22)$ Fi_{22} 64 561 751 654 400	$M(23)$ Fi_{23} 4 089 470 473 293 004 800	$F_{3+}, M(24)'$ Fi'_{24} 1 255 205 709 190 661 721 292 800	F_2 B 4 154 781 481 226 426 191 177 580 544 000 000	F_1, M_1 M 808 017 424 794 512 875 886 459 904 961 710 757 005 754 368 000 000 000	

http://irandrus.wordpress.com/2012/06/17/the-periodic-table-of-finite-simple-groups/

The Monster, a.k.a., the Friendly Giant

The largest of all the sporadic finite simple groups, it has order

$$2^{46} \cdot 3^{20} \cdot 5^{9} \cdot 7^{6} \cdot 11^{2} \cdot 13^{3} \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71$$

or

$$808017424794512875886459904961710757005754368000000000$$

which is approximately $8 \cdot 10^{53}$.