Farey Permutations

S. Bockting-Conrad, Y. Kashina, T. K. Petersen, B. Tenner

DePaul University

AMS - Hawaii

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   where $f(i) = \alpha i + \beta \pmod 1$
4. Sort this list with a permutation $\pi = \pi_{\alpha,\beta}$:

$$0 \leq f(\pi(0)) \leq f(\pi(1)) \leq \cdots \leq f(\pi(n)) < 1$$

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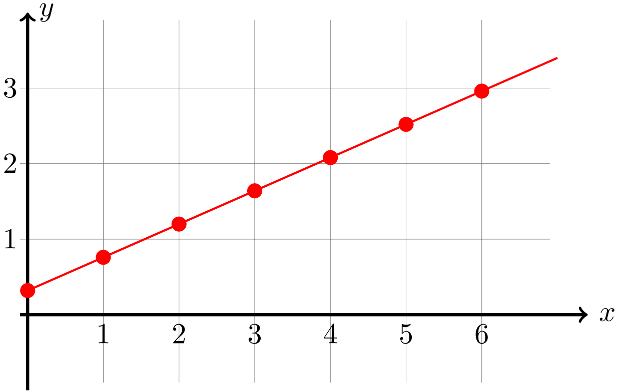

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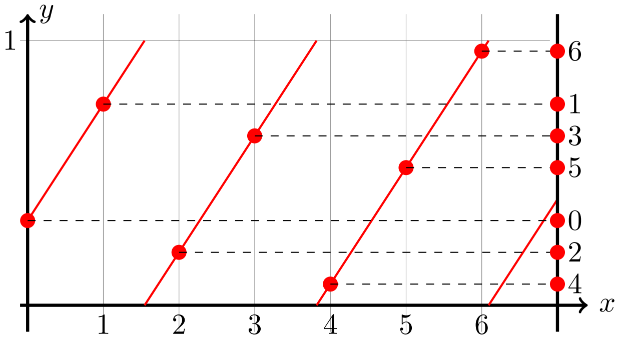

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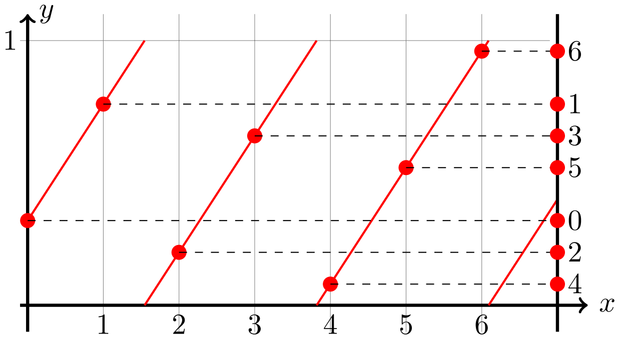

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A: We can say precisely

Enumerative Result

The number of Farey permutations of $\{0, 1, \ldots, n\}$, for $n = 0, 1, 2, \ldots$ begins

$$1, 2, 6, 16, 30, 60, 84, 144, 198, \ldots$$

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For $n \geq 1$, the number of Farey permutations is given by

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Proposition

For fixed α, let $\pi = \pi_{\alpha,0}$. Then

$$\{\pi_{\alpha,\beta} : 0 \leq \beta < 1\} = \{\pi \cdot c^k : k = 0, 1, \ldots, n\}$$

where c is the cycle $c = (n01 \cdots (n-1))$

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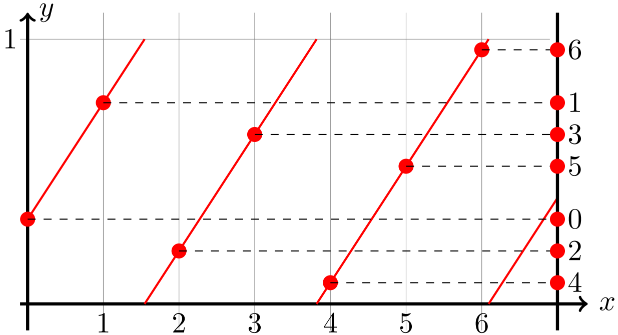

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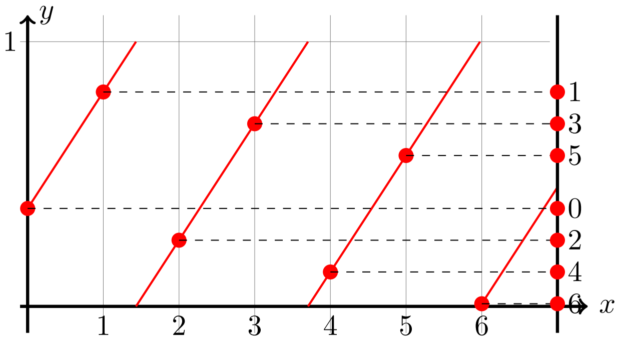

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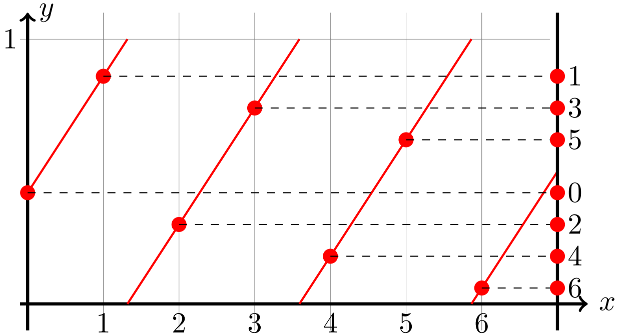

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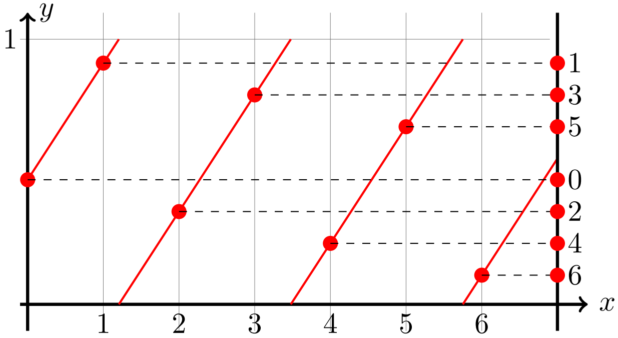

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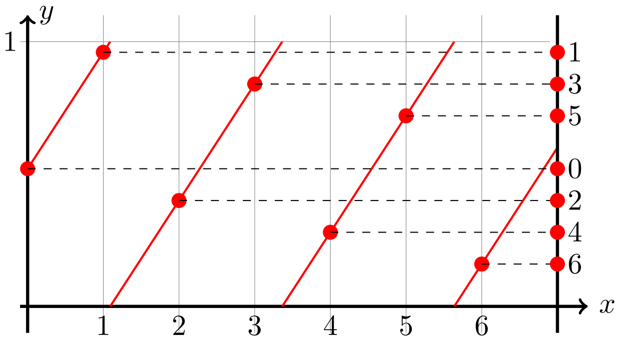

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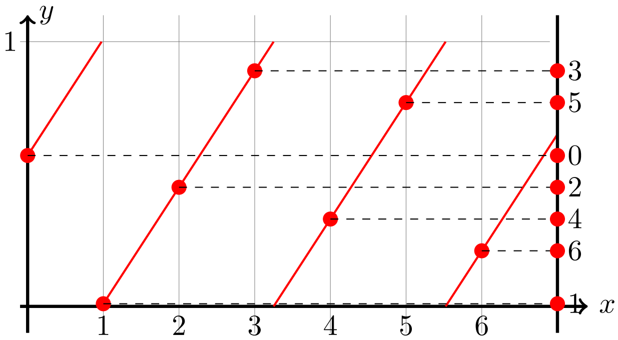

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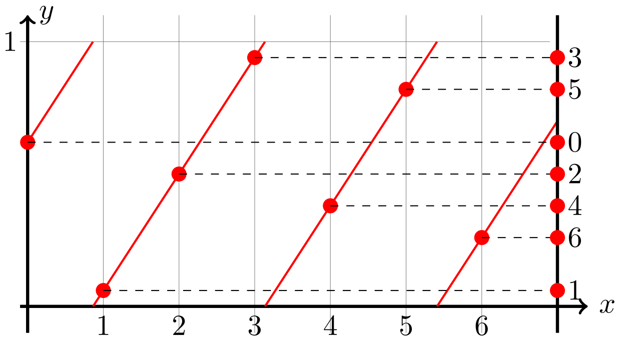


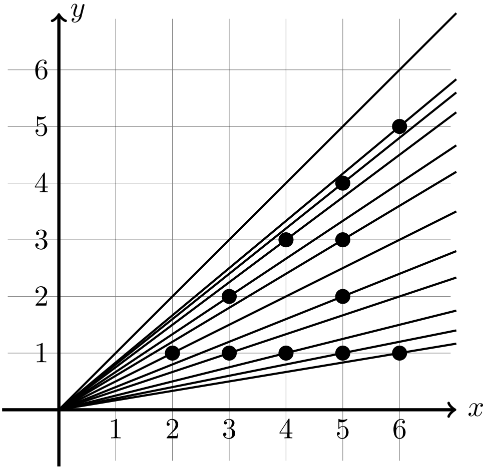
Results

The factor of

$$\varphi(1) + \varphi(2) + \cdots + \varphi(n)$$

is more interesting

Case of $\beta = 0$



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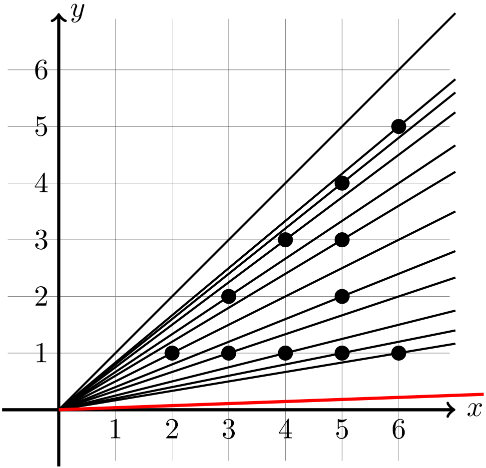

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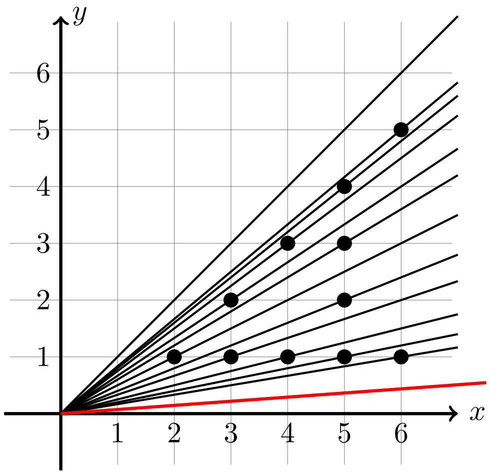

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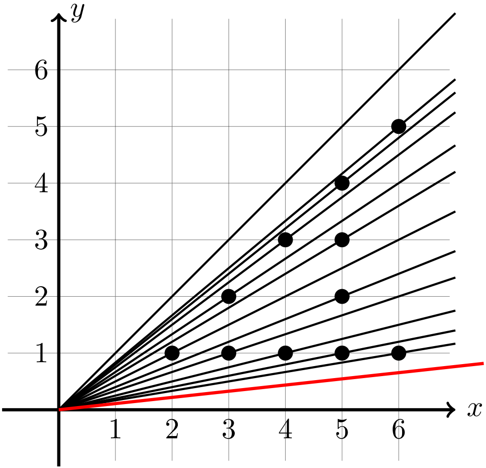

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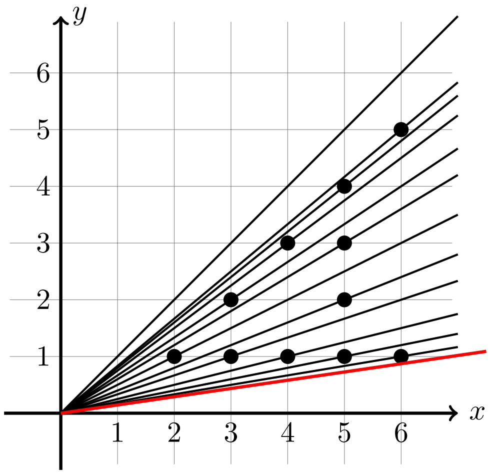

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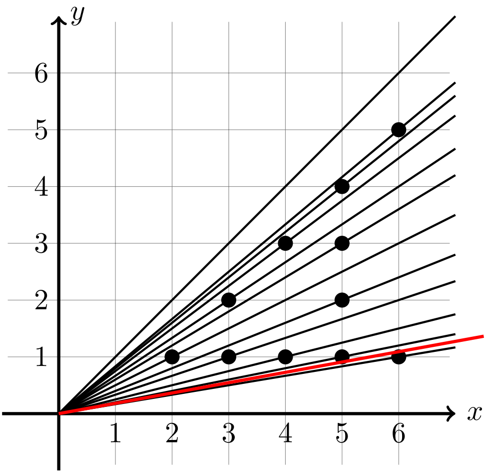

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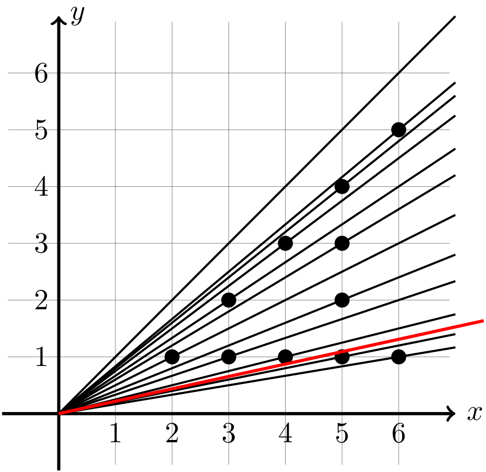

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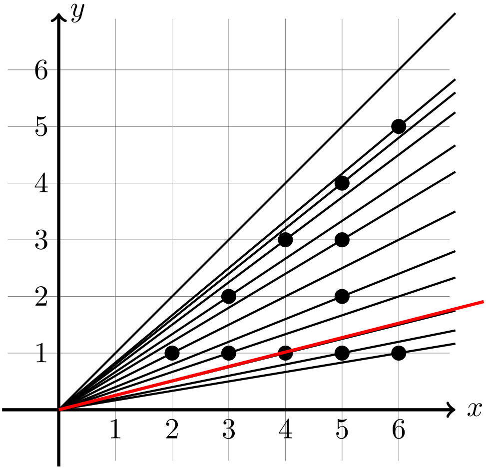

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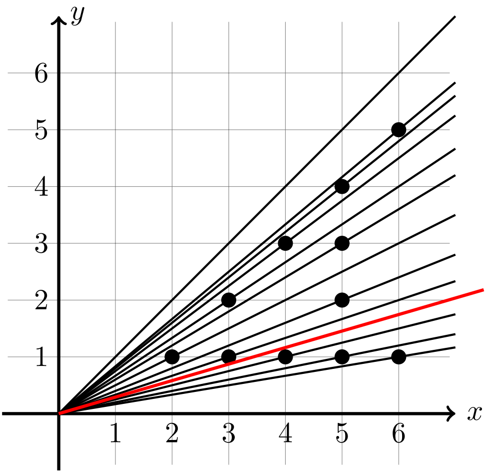

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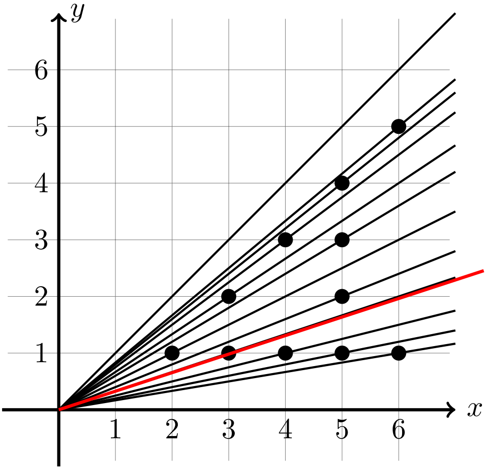

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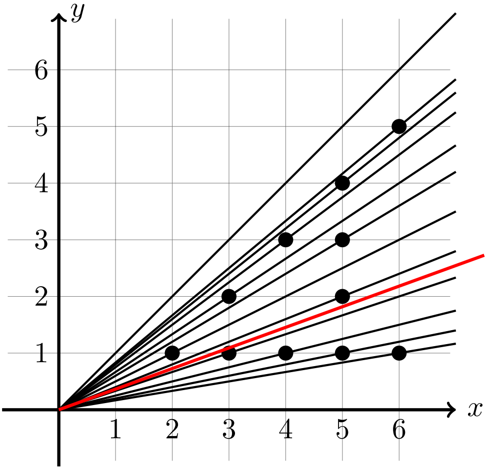

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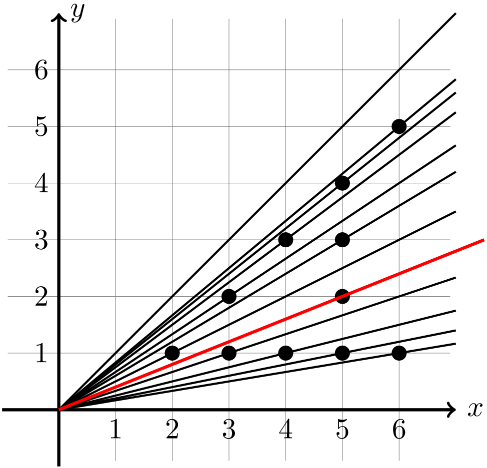

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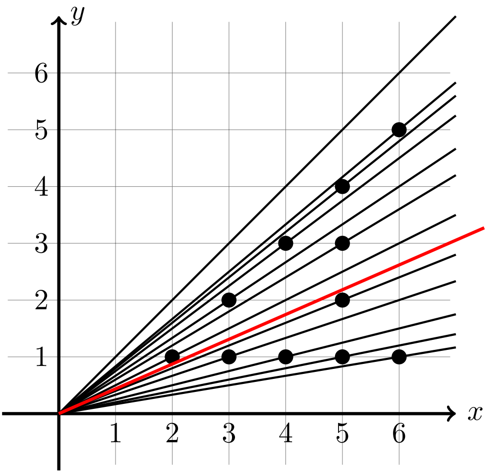

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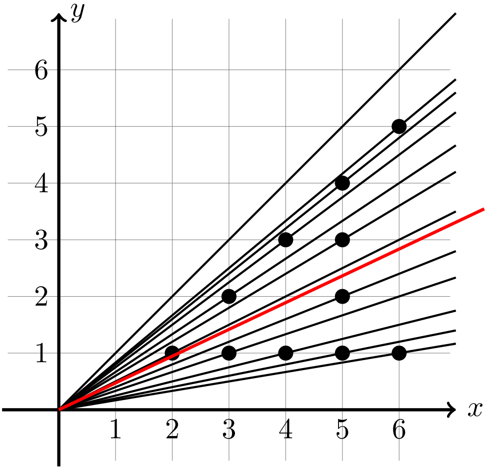

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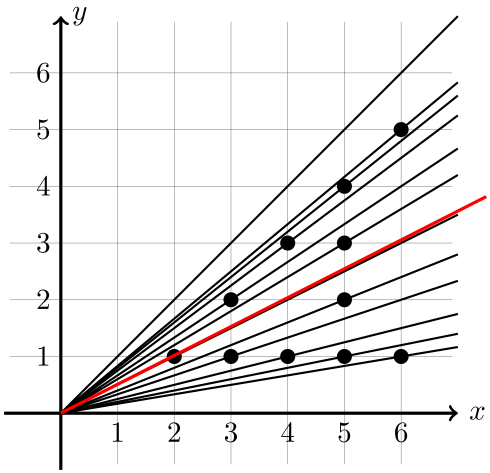

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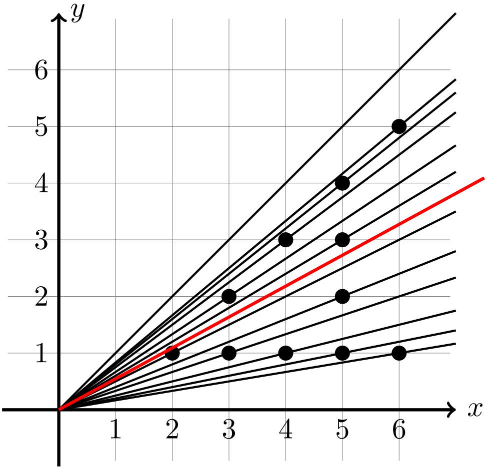

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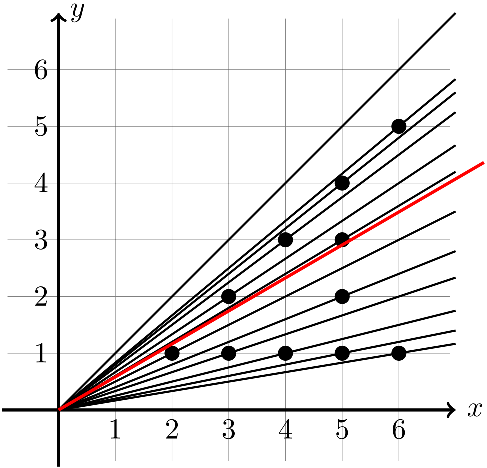

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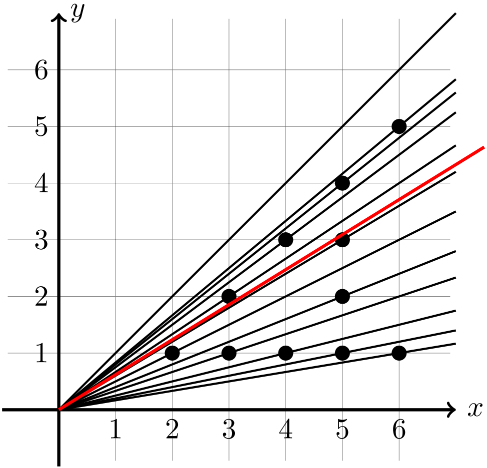

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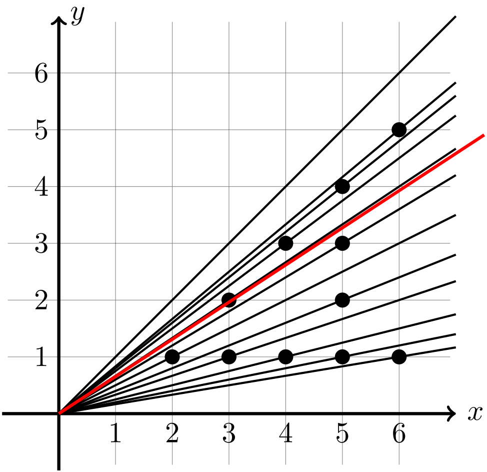

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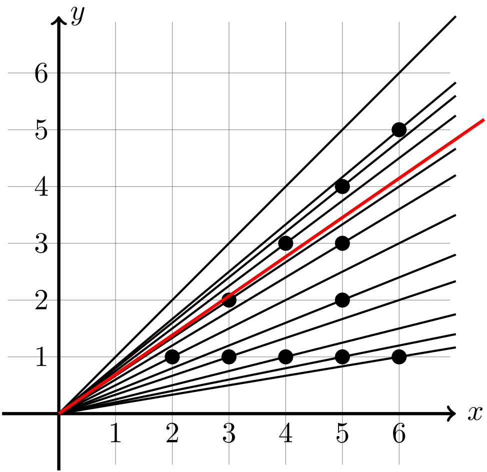

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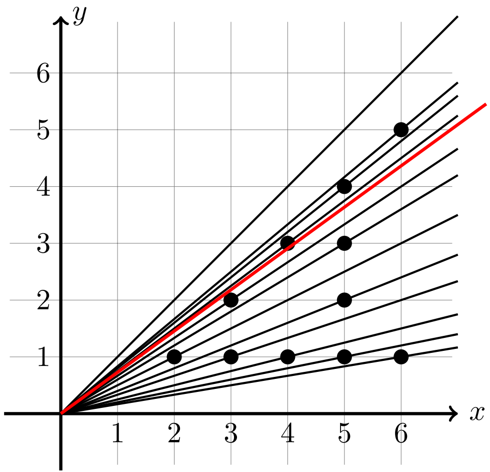

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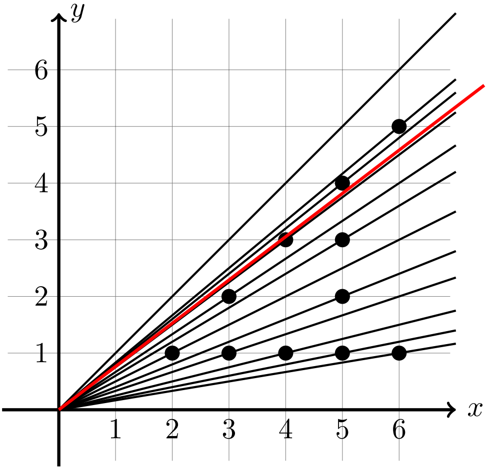

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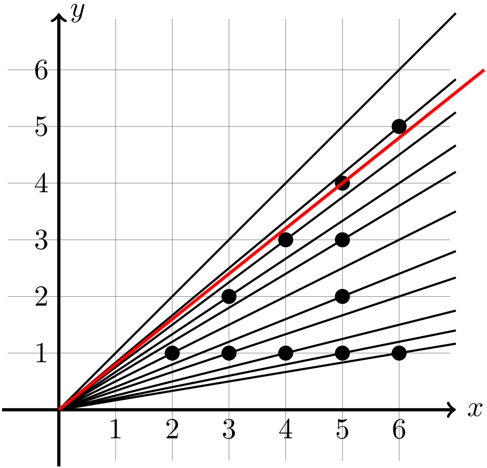

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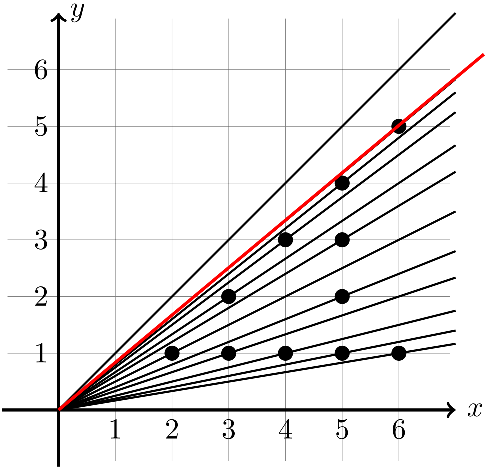

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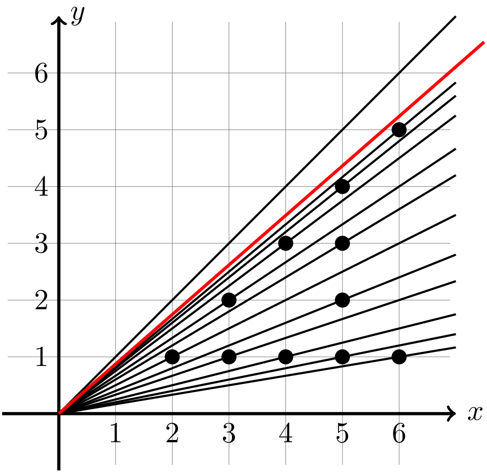

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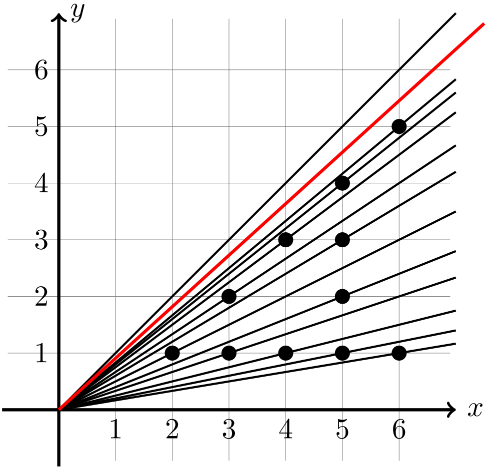

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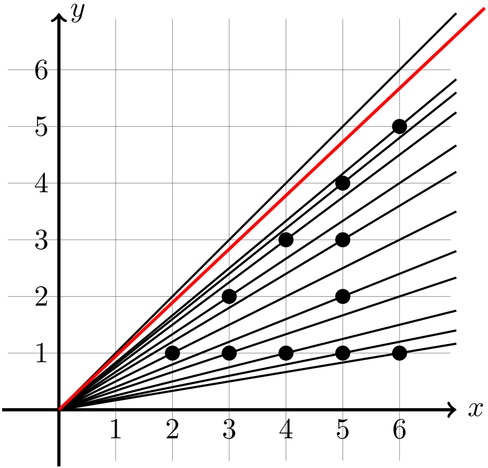

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Bijection with Farey intervals

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- $\pi(0) = 0$, $\pi(1) = b$, $\pi(n) = d$
- *for* $k \geq 1$,

$$\pi(k+1) = \begin{cases} \pi(k) + b & \text{if } \pi(k) < d \text{ and } b + \pi(k) \leq n, \\ \pi(k) - d & \text{if } \pi(k) > d \text{ and } b + \pi(k) > n \\ \pi(k) + b - d & \text{if } \pi(k) < d \text{ and } b + \pi(k) > n. \end{cases}$$

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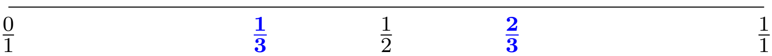

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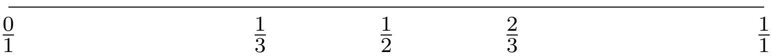

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Idea of proof via mediant approximation

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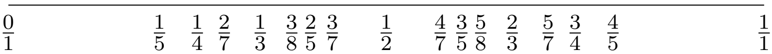

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So $\alpha \approx 4/9$

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now, $f(i) \approx 4i \pmod 9$,

$$[\ ,\ ,\ ,\ ,\ ,\ ,\ ,\ ,\ ,\qquad]$$

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$$[0, 4, 8, 3, 7, 2, \ , \ , \ , \qquad]$$

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entries 0 to 8 sort with

$$\pi = [0, 7, 5, 3, 1, 8, 6, 4, 2]$$

note two gap sizes: $b = 7$ and $d = 2$

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note two gap sizes: $b = 7$ and $d = 2$
if we wanted only $n = 7$, then delete 8 to get a gap of size $b - d = 5$:

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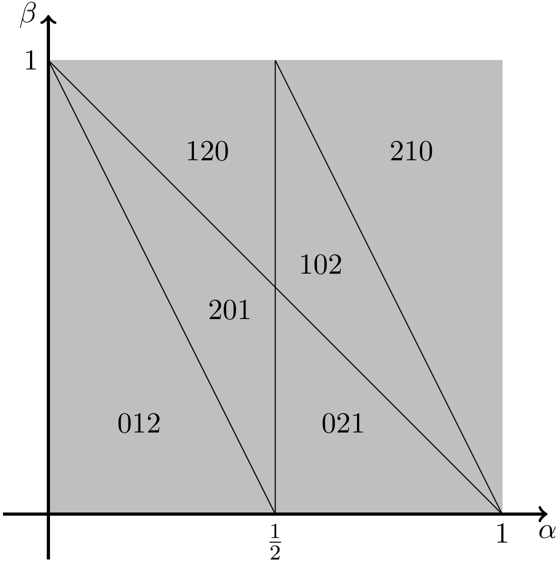
Back to the general case

We now try to consider **all** values of α and β that give the same permutation

Domains



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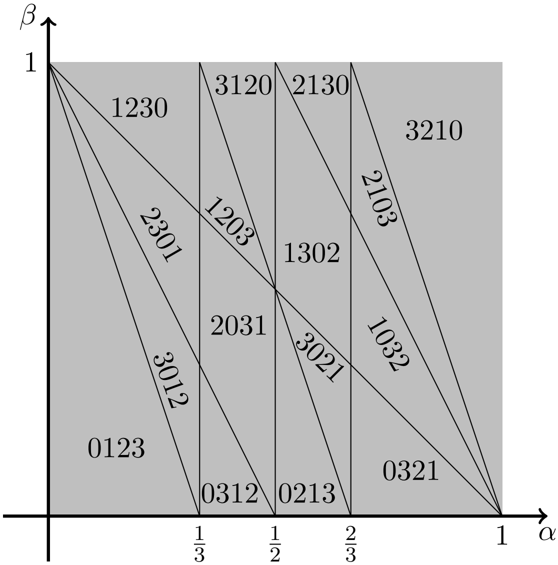
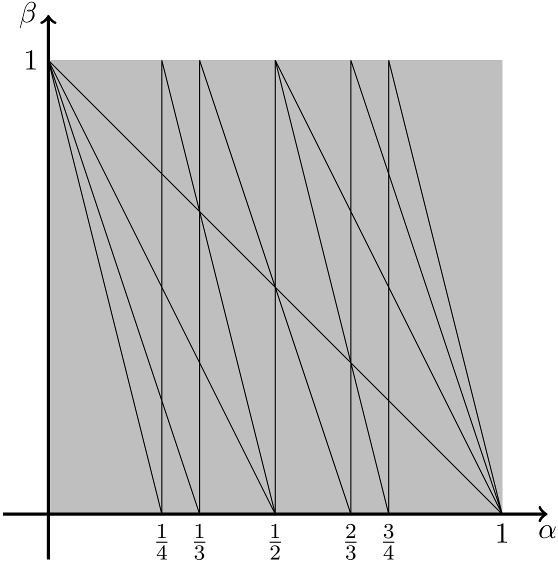
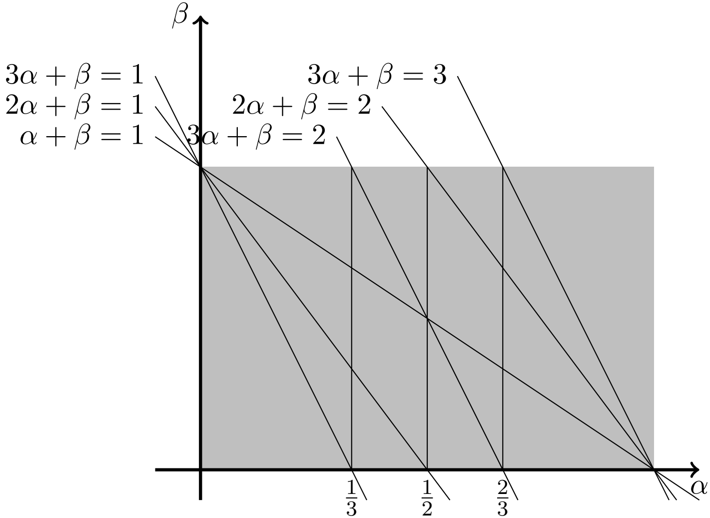


β
1
1230
3120
2130
3210
2301
1203
2103
1302
2031
3021
1032
3012
0123
0321
0312
0213
1/3
1/2
2/3
1
α

Domains



Domains



β
$3\alpha+\beta=1$
$2\alpha+\beta=1$
$\alpha+\beta=1$
$3\alpha+\beta=3$
$2\alpha+\beta=2$
$3\alpha+\beta=2$
α
$\frac{1}{3}$
$\frac{1}{2}$
$\frac{2}{3}$

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Theorem

Fix n and let π be a Farey permutation with $\pi(k) = 0$. Then $\pi = \pi_{\alpha,\beta}$ for all points α, β such that:

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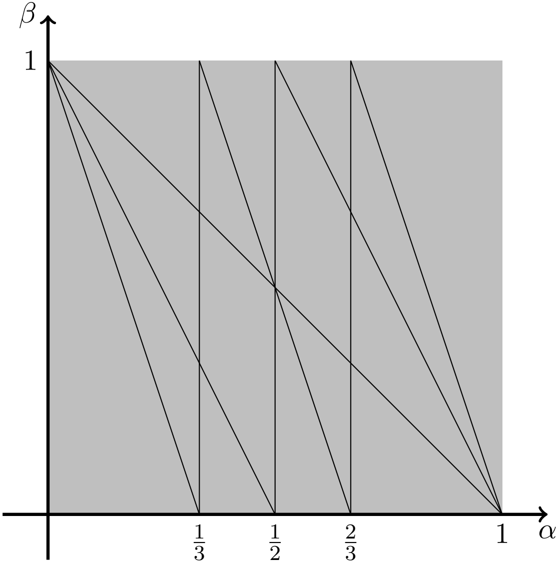
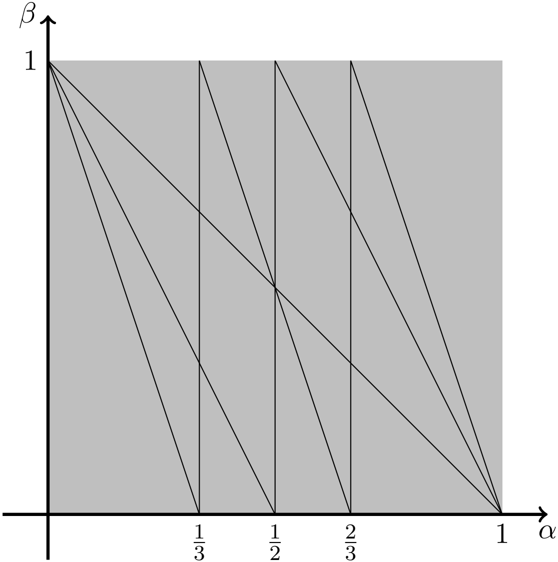
- *α is in the Farey interval $(\frac{a}{b}, \frac{c}{d})$, where $b = \pi(k+1)$ and $d = \pi(k-1)$ (which uniquely determines the interval)*
- *$\ell(0) < \beta < \ell(n)$, where $\ell(i)$ is the line defined by $\ell(i) = 1 + \lfloor \pi(i)\frac{a}{b} \rfloor - \pi(i)\alpha$*

Domains



Domains

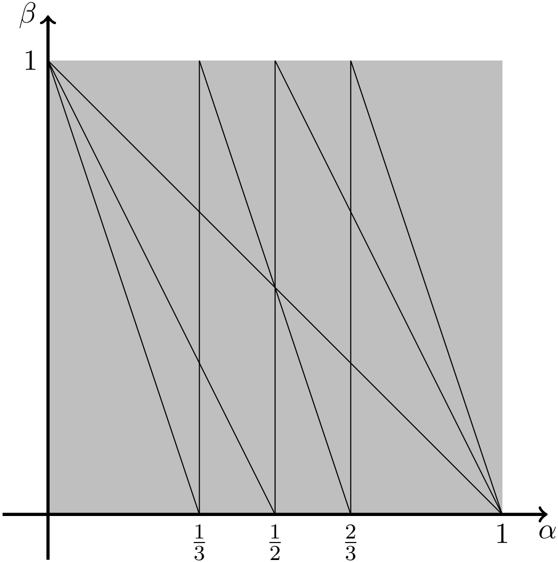
2031



Domains

2031

$b = 3, d = 2$



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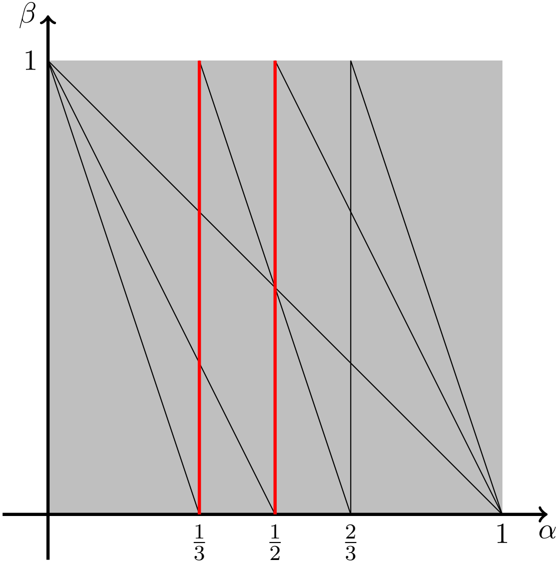

Domains

2031

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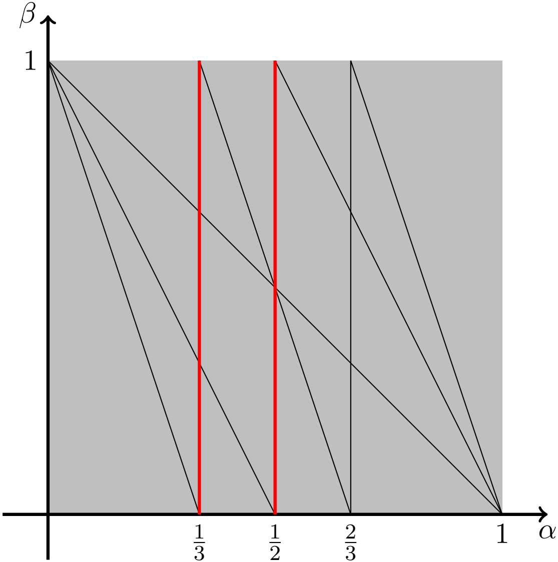

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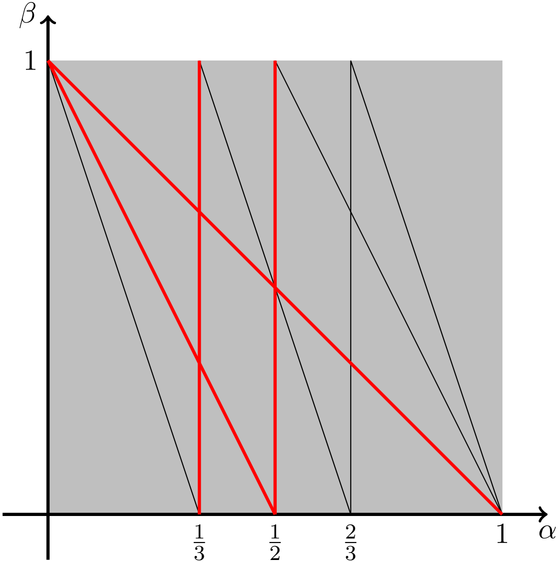

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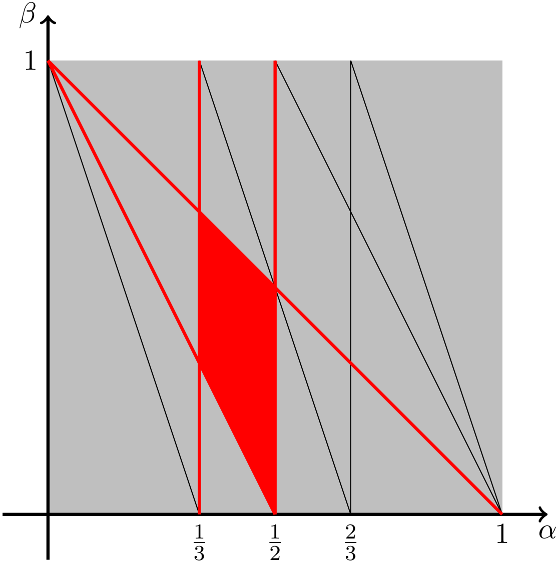

Motivation

Original motivation here came from thinking about pseudo-randomness, e.g., Elizalde's work with discrete dynamical systems ("β-shifts") and Steele's work with the sequences $\{\alpha n \pmod 1\}, n \to \infty$ (related to Ulam's problem of longest increasing subsequences)

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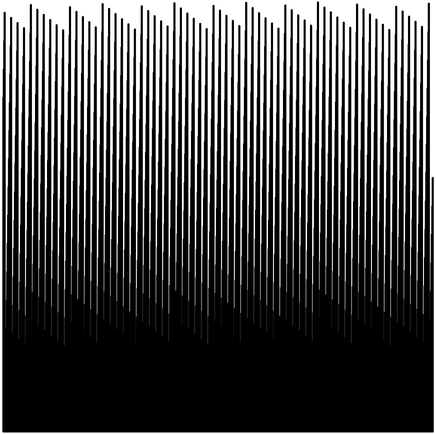
...the jury is still out, but

Random Farey permutations are not very random!



Thank you!