

ALTERNATIVE STABLE HOMOTOPY CLASSIFICATION OF BG_p^\wedge

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ABSTRACT. We give an alternative to the stable classification of p -completed homotopy types of classifying spaces of finite groups offered by Martino-Priddy in [8]. For a finite group G with Sylow subgroup S , we regard the stable p -completed classifying space $\Sigma^\infty BG_p^\wedge$ as an object under $\Sigma^\infty BS$ via the canonical inclusion map. Thus we get a classification in terms of induced fusion systems. Applying Oliver's solution [11, 12] to the Martino-Priddy conjecture [9], we obtain the surprising result that the unstable homotopy type of BG_p^\wedge is determined by the map $\Sigma^\infty BS \rightarrow \Sigma^\infty BG_p^\wedge$, but not by the homotopy type of $\Sigma^\infty BG_p^\wedge$.

INTRODUCTION

Using Carlsson's solution of the Segal Conjecture [4], and Nishida's theory of dominant summands [10], Martino-Priddy proved a classification theorem for stable homotopy types of p -completed classifying space of finite groups in [8]. For a set X , let $\mathbb{F}_p X$ denote the \mathbb{F}_p -vector space with basis X and for groups Q and G , let $\text{Rep}(Q, G) = \text{Hom}(Q, G)/G$ with G acting by conjugation, and let $\text{InjRep}(Q, G) \subset \text{Rep}(Q, G)$ be the set of conjugacy classes of injective homomorphisms.

Theorem (Stable classification [8]). *For two finite groups G and G' , the following are equivalent:*

- (1) BG_p^\wedge and BG'_p^\wedge are stably homotopy equivalent.
- (2) For every p -group Q ,

$$\mathbb{F}_p \text{Rep}(Q, G) \cong \mathbb{F}_p \text{Rep}(Q, G')$$

as $\text{Out}(Q)$ -modules.

- (3) For every p -group Q ,

$$\mathbb{F}_p \text{InjRep}(Q, G) \cong \mathbb{F}_p \text{InjRep}(Q, G')$$

as $\text{Out}(Q)$ -modules.

In this paper we take a different point of view. By regarding the stable p -completed classifying space $\mathbb{B}G_p^\wedge = \Sigma^\infty BG_p^\wedge$ of a finite group G as an object under the stable classifying space $\mathbb{B}S$ of its Sylow subgroup S and using the Segal conjecture, we are able to reconstruct the fusion

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system $\mathcal{F}_S(G)$. This is the category whose objects are the subgroups of S , and whose morphisms are the morphisms induced by conjugation in G . Thus we are able to classify the stable homotopy types $\Sigma^\infty BG_p^\wedge$ as objects under the stable classifying spaces of their Sylow subgroups by their induced fusion systems. By Oliver's solution of the Martino-Priddy conjecture [11, 12], the unstable homotopy types of p -completed classifying spaces of finite groups are also classified by their fusion systems, so we get the following theorem which is proved in Section 3, where the notion of an isomorphism of fusion systems is also explained.

Theorem (Alternative stable classification). *For two finite groups G and G' with Sylow subgroups S and S' , respectively, the following are equivalent:*

- (i) *There is an isomorphism $\gamma: S \rightarrow S'$ and a homotopy equivalence $h: \mathbb{B}G_p^\wedge \rightarrow \mathbb{B}G'_p^\wedge$ such that the following diagram commutes up to homotopy:*

$$\begin{array}{ccc} \mathbb{B}S & \xrightarrow{\mathbb{B}\iota_S} & \mathbb{B}G_p^\wedge \\ \mathbb{B}\gamma \downarrow & & \downarrow h \\ \mathbb{B}S' & \xrightarrow{\mathbb{B}\iota_{S'}} & \mathbb{B}G'_p^\wedge. \end{array}$$

- (ii) *There is an isomorphism of fusion systems*

$$(S, \mathcal{F}_S(G)) \longrightarrow (S', \mathcal{F}_{S'}(G')).$$

- (iii) *There is a homotopy equivalence of p -completed classifying spaces*

$$h: BG_p^\wedge \longrightarrow BG'_p^\wedge.$$

The presence of the third condition in this theorem is rather surprising, since Martino-Priddy have in [8, Example 5.2] constructed two groups whose p -completed classifying spaces are homotopy equivalent stably, but not unstably. The added datum of the map $\mathbb{B}S \rightarrow \mathbb{B}G$ therefore really gives new algebraic information. The cost of this added information is to only allow maps $\mathbb{B}S \rightarrow \mathbb{B}S'$ induced by group homomorphisms. This is an interesting point, which will be taken up in Section 4.

This paper is split into four sections. In Section 1 we recall some background material on the Segal conjecture and in Section 2 we do the same for Sylow subgroups of spaces. In Section 3 we prove the alternative stable classification theorem and in Section 4 we compare the two stable classification theorems to each other as well as to the Martino-Priddy conjecture.

Throughout this paper, p is a fixed prime. We denote the inclusion of a group H into a supergroup by ι_H , specifying the target group if it is not clear from the context. For a space or a spectrum X , let X_p^\wedge denote the Bousfield-Kan p -completion of X [2]. For spaces X and

Y , let $\{X, Y\}$ denote the group of homotopy classes of stable maps $\Sigma^\infty X \rightarrow \Sigma^\infty Y$. Unless otherwise specified, spaces and homotopies will be assumed to be unpointed. We use the shorthand notation $\mathbb{B}(-)$ for the functor $\Sigma^\infty B(-)$. Recall that $\Sigma^\infty(BG_p^\wedge) \simeq (\Sigma^\infty BG)_p^\wedge$ for a finite group G and we write $\mathbb{B}G_p^\wedge$ without danger of confusion. Finally, recall that p -completion coincides with p -localization for classifying spaces of finite groups, so we could equally well state all results for p -localizations.

1. BURNSIDE MODULES AND THE SEGAL CONJECTURE

For finite groups G and G' , we use the term (G, G') -biset to denote a set with a right G -action and a free left G' -action such that the two actions commute. Let $\text{Mor}(G, G')$ denote the set of isomorphism classes of (G, G') -bisets. The operation of taking disjoint unions provides an abelian monoid structure on $\text{Mor}(G, G')$. We refer to the Grothendieck group completion of $\text{Mor}(G, G')$ as the *Burnside module of G and G'* and denote it by $A(G, G')$. Being an abelian group, we can regard $A(G, G')$ as a \mathbb{Z} -module, and as such its structure is easy to describe.

Definition 1.1. *Let G and G' be finite groups. A (G, G') -pair is a pair (H, φ) consisting of a subgroup $H \leq G$ and a homomorphism $\varphi: H \rightarrow G'$. We say two (G, G') -pairs (H, φ) and (H', φ') are conjugate if there exist elements $g \in G$ and $h \in G'$ such that $c_g(H) = H'$ and the following diagram commutes:*

$$\begin{array}{ccc}
 H & \xrightarrow{\varphi} & G' \\
 \cong \downarrow c_g & & \downarrow c_h \\
 H' & \xrightarrow{\varphi'} & G'.
 \end{array}$$

Conjugacy is an equivalence relation on (G, G') -pairs and we denote the conjugacy class of a (G, G') -pair (H, φ) by $[H, \varphi]$. The Burnside module $A(G, G')$ can now be described as a free \mathbb{Z} -module with one basis element for each conjugacy class of (G, G') -pairs. By a slight abuse of notation we will denote the basis element of $A(G, G')$ corresponding to a conjugacy class $[H, \varphi]$ also by $[H, \varphi]$.

For a space X , let X_+ denote the pointed space obtained by adding a disjoint basepoint to X , and let $\Sigma_+^\infty X$ denote the suspension spectrum of X_+ . There is a homomorphism

$$\alpha: A(G, G') \longrightarrow \{BG_+, BG'_+\},$$

defined on basis elements by sending $[H, \varphi]$ to the map

$$\mathbb{B}\varphi \circ tr_H: \Sigma_+^\infty BG \xrightarrow{tr_H} \Sigma_+^\infty BH \xrightarrow{\Sigma_+^\infty B\varphi} \Sigma_+^\infty BG',$$

where tr_H is the transfer of the inclusion $H \hookrightarrow G$. The Segal conjecture, which was proved by Carlsson in [4], states that this homomorphism

is a completion with respect to a certain ideal when G' is the trivial group. Lewis-May-McClure showed in [6] that consequently the same holds for any finite group G' .

When the source group G is a p -group, this completion can be described in a particularly simple way; at least after getting rid of base-points. Let $\tilde{A}(G, G')$ denote the module obtained from $A(G, G')$ by quotienting out all basis elements of the form $[P, \varphi]$, where φ is the trivial homomorphism. Recalling that $\Sigma_+^\infty BG \simeq \Sigma^\infty BG \vee \mathbb{S}^0$, one can check that α induces a map

$$\tilde{A}(G, G') \longrightarrow \{BG_+, BG'_+\}/\{BG_+, S^0\} \cong \{BG, BG'\}.$$

Theorem 1.2 (Segal Conjecture [4, 6]). *Let S be a finite p -group and G be any finite group. Then the homomorphism α described above induces an isomorphism*

$$\tilde{\alpha}_p^\wedge: \tilde{A}(S, G)_p^\wedge \xrightarrow{\cong} \{BS, BG\},$$

where $(-)_p^\wedge = (-) \otimes \mathbb{Z}_p^\wedge$ is p -adic completion.

2. HOMOTOPY MONOMORPHISMS AND SUBGROUPS

In this section we describe a homotopy theoretic analogue of group monomorphisms and subgroup inclusions. The discussion is very goal-oriented and we develop just enough tools to work with Sylow subgroups of spaces, as defined below. The reader should be aware that there are more than one version of homotopy monomorphisms to be found in the literature and we have here selected one that is suitable for our purpose.

Definition 2.1. *A map $f : X \rightarrow Y$ of spaces is a homotopy monomorphism at p if the induced map in \mathbb{F}_p -cohomology makes $H^*(X; \mathbb{F}_p)$ a finitely generated $H^*(Y; \mathbb{F}_p)$ -module.*

As the prime p is fixed we will just say "homotopy monomorphisms" without danger of confusion. A motivation for the above definition is the following lemma, a proof of which can be found in [5].

Lemma 2.2. *Let $\varphi : P \rightarrow Q$ be a homomorphism of finite p -groups. Then φ is a group monomorphism if and only if $B\varphi$ is a homotopy monomorphism.*

The next lemma has an immediate algebraic proof, which is left to the reader.

Lemma 2.3. *Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be maps of spaces.*

- (i) *If f and g are homotopy monomorphisms then the composite $g \circ f$ is a homotopy monomorphism.*
- (ii) *If $g \circ f$ is a homotopy monomorphism then f is a homotopy monomorphism.*

We now arrive at the main purpose of this section.

Definition 2.4. A p -subgroup of a space X is a pair (P, f) where P is a finite p -group and $f: BP \rightarrow X$ is a homotopy monomorphism. A p -subgroup (S, f) of a space X is a Sylow p -subgroup of X if every map $g: BP \rightarrow X$, where P is a finite p -group, factors up to homotopy through $f: BS \rightarrow X$.

Example 2.5. Let G be a finite group with Sylow p -subgroup S and let ι_S be the inclusion $S \hookrightarrow G$. By Sylow's second theorem, any homomorphism $\varphi: P \rightarrow G$ factors through ι_S up to conjugacy. Recalling that the classifying space functor induces a bijection

$$\text{Rep}(P, G) \xrightarrow{\cong} [BP, BG],$$

we see that $(S, B\iota_S)$ is a Sylow p -subgroup of BG . Since the p -completion functor induces a bijection

$$[BP, BG] \xrightarrow{\cong} [BP, BG_p^\wedge],$$

(this is a well known result, a proof of which can be found for example in [3],) we see that $(S, (B\iota_S)_p^\wedge)$ is a Sylow p -subgroup of BG_p^\wedge .

The following lemma will be needed in the proof of the main theorem of this paper and also to explain the difference between the two stable classifications, as well as the difference between the stable and unstable classifications.

Lemma 2.6. Let X be a space and let (S, f) and (S', f') be two Sylow subgroups of X . Then there exists an isomorphism $\gamma: S \xrightarrow{\cong} S'$ such that $f' \circ B\gamma \simeq f$.

Proof. By the Sylow property of (S', f') , there is a map $h: BS \rightarrow BS'$ such that $f' \circ h \simeq f$. Since f is a homotopy monomorphism, h must be a homotopy monomorphism. Pick a $\gamma \in \text{Hom}(S, S')$ such that $h \simeq B\gamma$. Then γ is a group monomorphism $S \rightarrow S'$ such that $f' \circ B\gamma \simeq f$. Similarly we get a monomorphism $S' \rightarrow S$. We deduce that S and S' have the same order and therefore γ is an isomorphism. \square

3. ALTERNATIVE STABLE CLASSIFICATION

In this section we prove the alternative stable classification theorem stated in the introduction. We begin by recalling the definition of fusion systems, which occur in the statement.

Definition 3.1. Let G be a finite group and let S be a Sylow subgroup of G . The fusion system of G over S is the category $\mathcal{F}_S(G)$ whose objects are the subgroups of S , and whose morphisms are the morphisms induced by conjugation in G :

$$\text{Hom}_{\mathcal{F}_S(G)}(P, Q) = \text{Hom}_G(P, Q).$$

An isomorphism of fusion systems $\mathcal{F}_S(G)$ and $\mathcal{F}_{S'}(G')$ is a group isomorphism $\gamma: S \xrightarrow{\cong} S'$ such that for all homomorphisms $\varphi: P \rightarrow Q$ between subgroups of S , we have $\varphi \in \mathcal{F}_S(G)$ if and only if $\gamma \circ \varphi \circ \gamma^{-1} \in \mathcal{F}_{S'}(G')$.

Martino-Priddy showed in [9] that the fusion system $\mathcal{F}_S(G)$ can be recovered from the inclusion $BS \rightarrow BG_p^\wedge$ via a simple homotopy theoretic construction. The following proposition shows that the same construction works even after infinite suspensions.

Proposition 3.2. *Let G be a finite group and let S be a Sylow subgroup of G . For subgroups P and Q of S we have*

$$\text{Hom}_{\mathcal{F}_S(G)}(P, Q) = \{\varphi \in \text{Hom}(P, Q) \mid \mathbb{B}\iota_Q \circ \mathbb{B}\varphi \simeq \mathbb{B}\iota_P: \mathbb{B}P \rightarrow \mathbb{B}G_p^\wedge\},$$

where ι_P and ι_Q denote the inclusions of P and Q in G .

Proof. Let $\varphi \in \text{Hom}(P, Q)$. By the Segal Conjecture, $\mathbb{B}\iota_Q \circ \mathbb{B}\varphi \simeq \mathbb{B}\iota_P$ if and only if $[P, \iota_Q \circ \varphi] = [P, \iota_P]$ in $\tilde{A}(P, G)_p^\wedge$, which is clearly the case if and only if $[P, \iota_Q \circ \varphi] = [P, \iota_P]$ in $A(P, G)$. By definition, the last equality means that there exist $g \in P$ and $h \in G$ such that the following diagram commutes:

$$\begin{array}{ccc} P & \xrightarrow{\iota_Q \circ \varphi} & G \\ \cong \downarrow c_g & & \downarrow c_h \\ P & \xrightarrow{\iota_P} & G, \end{array}$$

or in other words such that

$$\varphi(x) = c_{h^{-1}g}(x)$$

for all $x \in P$. This is in turn true if and only if φ is induced by a conjugation in G . \square

Using Proposition 3.2, and Oliver's solution of the Martino-Priddy conjecture, the alternative stable classification theorem stated in the introduction follows easily.

Proof of alternative stable classification. Put

$$\mathcal{F}_1 := \mathcal{F}_S(G), \quad \text{and} \quad \mathcal{F}_2 := \mathcal{F}_{S'}(G').$$

(i) \Rightarrow (ii) : We show that γ is an isomorphism of fusion systems. For subgroups $P, Q \leq S$, and a homomorphism $\varphi \in \text{Hom}_{\mathcal{F}_1}(P, Q)$, we have

$$\begin{aligned} \mathbb{B}\iota_{\gamma(Q)} \circ \mathbb{B}(\gamma|_Q \circ \varphi \circ \gamma^{-1}|_{\gamma(P)}) &\simeq h \circ \mathbb{B}\iota_Q \circ \mathbb{B}\varphi \circ \mathbb{B}\gamma^{-1}|_{\gamma(P)} \\ &\simeq h \circ \mathbb{B}\iota_P \circ \mathbb{B}\gamma^{-1}|_{\gamma(P)} \\ &\simeq \mathbb{B}\gamma|_P \circ \mathbb{B}\gamma^{-1}|_{\gamma(P)} \\ &\simeq \mathbb{B}\iota_{\gamma(P)}, \end{aligned}$$

so

$$\gamma|_Q \circ \varphi \circ \gamma^{-1}|_{\gamma(P)} \in \text{Hom}_{\mathcal{F}_2}(\gamma(P), \gamma(Q)).$$

By symmetry we see that if $\gamma|_Q \circ \varphi \circ \gamma^{-1}|_{\gamma(P)} \in \text{Hom}_{\mathcal{F}_2}(\gamma(P), \gamma(Q))$, then $\varphi \in \text{Hom}_{\mathcal{F}_1}(P, Q)$.

(ii) \Rightarrow (iii) : This follows from Oliver's proof of the Martino-Priddy conjecture [11, 12].

(iii) \Rightarrow (i) : Since $(S', B\iota_{S'})$ and $(S, h \circ B\iota_S)$ are both Sylow p -subgroups of BG_p^\wedge , there is by Lemma 2.6 an isomorphism $\gamma: S \rightarrow S'$ making the following diagram commute up to homotopy:

$$\begin{array}{ccc} BS & \xrightarrow{B\iota_S} & BG_p^\wedge \\ B\gamma \downarrow & & \downarrow h \\ BS' & \xrightarrow{B\iota_{S'}} & BG_p'^\wedge. \end{array}$$

(i) follows upon infinite suspension. □

4. COMPARISON OF STABLE CLASSIFICATIONS

It is interesting to compare these two stable classifications. It is easy to see that Condition (i) in the alternative stable classification implies Condition (1) in the Martino-Priddy classification stated in the introduction, and that Condition (ii) implies Conditions (2) and (3). However, the reverse implications are not true. In [8, Example 5.2] Martino-Priddy construct groups G and G' , whose p -completed classifying spaces are equivalent stably, but not unstably and it is not difficult to see that their induced fusion systems are non-isomorphic. Therefore the alternative classification theorem is neither stronger nor weaker than the Martino-Priddy classification theorem (in the sense that one cannot be deduced from the other), but it does offer a 'finer' classification of p -completed stable classifying spaces of finite groups by keeping track of more structure.

Recall that for a finite group G with Sylow subgroup S , a simple transfer argument shows that $\mathbb{B}G_p^\wedge$ is a wedge summand of $\mathbb{B}S$. In their classification, Martino-Priddy regard $\mathbb{B}G_p^\wedge$ as a wedge sum of indecomposable stable summands of $\mathbb{B}S$. Thus two finite groups G and G' , both with Sylow subgroups isomorphic to a finite p -group S , have stably homotopy equivalent p -completed classifying spaces if and only if the multiplicity of each indecomposable wedge summand of $\mathbb{B}S$ is the same in $\mathbb{B}G_p^\wedge$ as in $\mathbb{B}G_p'^\wedge$. Our new point of view is to differentiate between the summands of $\mathbb{B}S$ and not just consider their homotopy types. By adding the data of the maps $\mathbb{B}S \rightarrow \mathbb{B}G_p^\wedge$ and $\mathbb{B}S \rightarrow \mathbb{B}G_p'^\wedge$, we obtain information about how the summands of $\mathbb{B}G_p^\wedge$ and $\mathbb{B}G_p'^\wedge$ 'sit inside' $\mathbb{B}S$. Instead of comparing the multiplicities of homotopy types of summands, we compare whether $\mathbb{B}G_p^\wedge$ contains the same summands of $\mathbb{B}S$ as $\mathbb{B}G_p'^\wedge$ does.

The price of differentiating between summands is to restrict the allowable maps between stable classifying spaces. For finite groups

G and G' , with respective Sylow subgroups S and S' , any map $h: \mathbb{B}G_p^\wedge \rightarrow \mathbb{B}G'_p^\wedge$ can be extended to a map $\bar{h}: \mathbb{B}S \rightarrow \mathbb{B}S'$ making the following diagram commute up to homotopy:

$$\begin{array}{ccc} \mathbb{B}S & \longrightarrow & \mathbb{B}G_p^\wedge \\ \bar{h} \downarrow & & \downarrow h \\ \mathbb{B}S' & \longrightarrow & \mathbb{B}G'_p^\wedge. \end{array}$$

The reason is that $\mathbb{B}G_p^\wedge$ is a wedge summand of $\mathbb{B}S$. It is by demanding in Condition (i) that the extension \bar{h} be induced by a group homomorphism $S \rightarrow S'$ that we restrict the allowable homotopy types of h . This way homotopy equivalent summands of $\mathbb{B}S$ can play different roles, based on the group structure of S . It could be interesting to reexamine the complete stable splittings of classifying spaces of finite groups in [1] and [7] from this point of view, and attempt to label the stable summands of the classifying space of a finite p -group in a way which takes into account their role with respect to maps induced by group homomorphisms.

It is also interesting that in Condition (iii) of the alternative stable classification theorem, we apparently do not regard BG_p^\wedge and BG'_p^\wedge as spaces under their Sylow subgroups. By Lemma 2.6 the homotopy type of BG_p^\wedge determines not only the isomorphism class of its Sylow subgroup S , but also the homotopy type of the pair $(BS, B\iota_S)$ as an object over BG_p^\wedge , so the homotopy type of BG_p^\wedge as an object under BS is implicit in the homotopy type of BG_p^\wedge .

In the stable setting, Nishida proved in [10] that the stable homotopy type of BG_p^\wedge determines the isomorphism class of the Sylow subgroup S . Since $\mathbb{B}G_p^\wedge$ is a wedge summand of $\mathbb{B}S$ and the map $\mathbb{B}\iota_S: \mathbb{B}S \rightarrow \mathbb{B}G_p^\wedge$ is a projection onto this summand, the homotopy type of the pair $(\mathbb{B}S, \mathbb{B}\iota_S)$ as an object over $\mathbb{B}G_p^\wedge$ is also determined. Therefore the homotopy type of $\mathbb{B}G_p^\wedge$ as an object under $\mathbb{B}S$ is also implicit in the homotopy type of $\mathbb{B}G_p^\wedge$.

The difference between the stable and unstable cases is that in the unstable case an equivalence between classifying spaces of Sylow subgroups $(BS, B\iota_S)$ and $(BS', B\iota_{S'})$ is a homotopy equivalence $h: BS \xrightarrow{\cong} BS'$ respecting the inclusion maps (up to homotopy) and this map must be induced by a group isomorphism. In the stable case, an equivalence between stable classifying spaces of Sylow subgroups $(\mathbb{B}S, \mathbb{B}\iota_S)$ and $(\mathbb{B}S', \mathbb{B}\iota_{S'})$ is similarly a homotopy equivalence $h: \mathbb{B}S \xrightarrow{\cong} \mathbb{B}S'$ respecting the inclusion maps (up to homotopy), but this map need not at all be induced by a group isomorphism (although by [10] its existence does imply that the groups are isomorphic). Therefore we need to impose a condition on the allowable maps between

stable classifying space of finite p -groups which is automatically satisfied for the unstable classifying spaces.

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