

**Math 217 - Spring 2014**  
**Quiz 1 Solutions**

1. (7 points) Perform Gauss Jordan elimination on the augmented matrix to solve the following linear system

$$\begin{array}{rrcr} & 2x_2 & + & x_3 & = & 2 \\ x_1 & + & 3x_2 & + & 2x_3 & = & -1 \\ 4x_1 & + & 6x_2 & + & 5x_3 & = & 3 \end{array}$$

**Solution :** The augmented matrix  $\left[ \begin{array}{ccc|c} 0 & 2 & 1 & 2 \\ 1 & 3 & 2 & -1 \\ 4 & 6 & 5 & 3 \end{array} \right]$  has reduced row-echelon form  $\left[ \begin{array}{ccc|c} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$ .

The original linear system is equivalent to a linear system with the third equation  $0 = 1$ . Therefore it must be inconsistent, that is, it has no solutions.

2. (4 points) Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  such that

$$T(\mathbf{x}) = \begin{bmatrix} -2x_1 + 5x_2 \\ 0 \\ 7x_2 \end{bmatrix}$$

Show that  $T$  is a linear transformation and find its matrix.

**Solution :** Let

$$A = \begin{bmatrix} -2 & 5 \\ 0 & 0 \\ 0 & 7 \end{bmatrix}.$$

Then  $T(\mathbf{x}) = A\mathbf{x}$  for all  $\mathbf{x} \in \mathbb{R}^2$ . By definition,  $T$  is a linear transformation and  $A$  is its matrix.

3. (9 points) In each part below is the augmented matrix of a linear system. First, determine whether each augmented matrix is in reduced-row echelon form. Then find all the solutions of the linear system and clearly state how many solutions are possible for each linear system.

(a)  $\left[ \begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$

**Solution :** The augmented matrix is in reduced row-echelon form.

The second row corresponds to the equation  $0 = 1$ , so there are no solutions.

(b)  $\left[ \begin{array}{cccc|c} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right]$

**Solution :** The augmented matrix is in reduced row-echelon form.

Since  $x_1$  is a free variable, there are infinitely many solutions :  $x_1 \in \mathbb{R}$ ,  $x_2 = x_3 = x_4 = 1$ .

(c)  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$

**Solution :** The augmented matrix is not in reduced row-echelon form because the fourth row has a pivot while a row above it, namely the third row, does not.

The system has the unique solution  $x_1 = x_2 = x_3 = 0$ .