MATH 217: LINEAR ALGEBRA SPRING 2104
WRITTEN HOMEWORK #8

Due Friday, June 6 at the beginning of class

Write clear and neat solutions to Problems 1 and 2 below and the following 6 problems from the textbook.

Problem 1 : Let V and W be linear spaces and let $T : V \to W$ be an isomorphism. Prove that $f_1, \dots, f_m \in V$ are linearly independent if and only if $T(f_1), \dots, T(f_m) \in W$ are linearly independent.

Problem 2 : Let V be a linear space of dimension n, and let $T : V \to V$ be a linear transformation. Let $\mathcal{B}$ be a basis for V, and assume $m \leq n$.

(a) Prove $\mathcal{F} = \{f_1, \dots, f_m\}$ is a basis for $\ker(T)$ if and only if $\{[f_1]_\mathcal{B}, \dots, [f_m]_\mathcal{B}\}$ is a basis for $\ker([T]_\mathcal{B})$. Here $[T]_\mathcal{B}$ is the $\mathcal{B}$-matrix for the transformation T, i.e., the $n \times n$ matrix such that

$$[Tf]_\mathcal{B} = [T]_\mathcal{B}[f]_\mathcal{B}, \qquad \forall f \in V.$$

Conclude that $\text{nullity}(T) = \text{nullity}([T]_\mathcal{B})$.

(b) Prove $\mathcal{F} = \{f_1, \dots, f_m\}$ is a basis for $\text{im}(T)$ if and only if $\{[f_1]_\mathcal{B}, \dots, [f_m]_\mathcal{B}\}$ is a basis for $\text{im}([T]_\mathcal{B})$. Show that $\text{rank}(T) = \text{rank}([T]_\mathcal{B})$.

Hint : Consider the coordinate transformation $L_\mathcal{B}$ and use Problem 1.

Section 4.3: Problems 59, 69 (Hint for 59 : Think Calculus)

Section 5.1: Problems 12, 22, 23, 25