MATH 255 **Applied Honors Calculus III** **Winter 2011**

Homework 3

Section 13.5, pg. 865: 2, 6, 22, 30, 38, 68.
Section 13.6, pg. 873: 30, 34, 46, 48.
Section 13.7, pg. 878: 46, 50, 58, 68.

Solutions

13.5: #2

For this line we have $\mathbf{r}_0 =< 1, 0, 3 >$ and $\mathbf{v} =< 2, -4, 5 >$. A vector equation is

$$\mathbf{r} = \mathbf{r}_0 + t\mathbf{v} =< 1, 0, 3 > +t < 2, -4, 5 >$$

and parametric equations are

$$x = 1 + 2t, \quad y = 0 - 4t, \quad z = 3 + 5t.$$

13.5: #6

This line is parallel to the vector $< 1, 2, 3 >$, and contains the origin $(0, 0, 0)$. Therefore we can write parametric equations

$$x = 0 + 1t, \quad y = 0 + 2t, \quad x = 0 + 3t$$

and symmetric equations

$$\frac{x-0}{1} = \frac{y-0}{2} = \frac{z-0}{3}.$$

13.5: #22

The lines are not parallel since the direction vectors $< 2, 2, -1 >$ and $< 1, -1, 3 >$ are not parallel (the ratio of components are not the same). To find if they intersect, we first solve for x, y and z from the symmetric equation for L_1:

$$\frac{x-1}{2} = t, \quad \frac{y-3}{2} = t, \quad \frac{z-2}{-1} = t$$

so the parametric equations are

$$x = 2t + 1, \quad y = 2t + 3, \quad z = -t + 2.$$

Similarly, the parametric equations for L_2 are:

$$x = t + 2, \quad y = -t + 6, \quad z = 3t - 2.$$

We now look for t_1 and t_2 that solve the system of equations

$$2t_1 + 1 = t_2 + 2\,, \quad 2t_1 + 3 = -t_2 + 6\,, \quad -t_1 + 2 = 3t_2 - 2.$$

According to the first equation, we should have $t_2 = 2t_1 - 1$. Plugging into the second equation, we get

$$2t_1 + 3 = -(2t_1 - 1) + 6\,,$$

and thus $t_1 = 1$ which gives $t_2 = 2(1) - 1 = 1$. Thus $t_1 = t_2 = 1$ solves the first two equations. It is easy to check that it solves the third equation as well, thus the lines do intersect. The point of intersection is $(x(1), y(1), z(1)) = (3, 5, 1)$.

13.5: #30

Since the plane is parallel to the plane $2x+4y+8z = 17$, we can use $< 2, 4, 8 >$ as its orthogonal vector. Also the plane contains the line $x = 3 + 2t$, $y = t$, $z = 8 - t$, therefore, we can use any point on that line. For example, for $t = 0$, the corresponding point is $(3, 0, 8)$. An equation of the plane is

$$2(x - 3) + 4(y - 0) + 8(z - 8) = 0.$$

13.5: #38

Let's first find the direction vector of the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$. Their normals are $\mathbf{n}_1 =< 1, 0, -1 >$ and $\mathbf{n}_2 =< 0, 1, 2 >$. The direction vector for the line is $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 =< 1, -2, 1 >$. The vector $\mathbf{v} =< 1, -2, 1 >$ is parallel to the plane in question. Another such vector is the normal vector $\mathbf{n}_3 =< 1, 1, -2 >$ for the plane $x + y - 2z = 1$. Hence the normal vector of the desired plane is

$$\mathbf{n} = \mathbf{v} \times \mathbf{n}_3 =< 1, -2, 1 > \times < 1, 1, -2 >=< 3, 3, 3 > .$$

To find a point on the plane we find a point on the line. For example, let $x = 0$. Then $z = -1$ and $y = 5$. We obtain the point $(0, -5, -1)$.
Finally, an equation of the plane is :

$$3(x - 0) + 3(y - 5) + 3(z + 1) = 0$$

or

$$x + y + z - 4 = 0.$$

13.5: #68

To find the distance, we can pick a point on the first plane, and find its distance to the second plane. Setting $x = 0, y = 0$ and plugging this in the first plane equation $3x + 6y - 9z = 4$ we have $z = \frac{-4}{9}$. So the point is $P(0, 0, \frac{-4}{9})$. The distance from P to the second plane is

$$\frac{|1 \cdot 0 + 2 \cdot 0 - 3 \cdot (\frac{-4}{9}) - 1|}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{1}{3\sqrt{14}}.$$

13.6: #30

The equation $x^2 = 2y^2 + 3z^2$ can be written as

$$\frac{x^2}{6} = \frac{y^2}{3} + \frac{z^2}{2}.$$

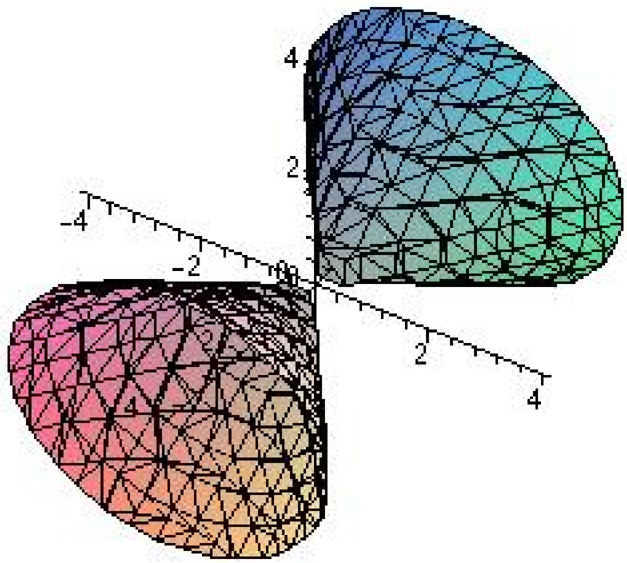

Figure 1: Plot for #30.

This is a cone with vertex at $(0, 0, 0)$. Its axis of symmetry is the x-axis (see fig 1).

13.6: #34

Completing the squares gives $4(y-2)^2+(z-2)^2 = x$. This is an elliptic paraboloid with vertex $(0, 2, 2)$ and axis the horizontal line $y = 2, z = 2$. See figure 2

13.6: #38

See figure 3

13.6: #40

See figure 4

13.6: #46

Suppose $P : (x, y, z)$ is a point on the desired surface. Then the distance from P to $y-z$ plane is x and to x-axis is $\sqrt{y^2 + z^2}$. By the description from the problem, we have $\sqrt{y^2 + z^2} = 2x$, that is $y^2 + z^2 = 4x^2$. It is a cone with x-axis as symmetric axis.

13.6: #48

The points on the curve satisfy both equations. From the first equation, by multiplying every term by 2, we can have $2x^2 + 4y^2 - z^2 = 2 - 6x$. Plug it into the second equation, we have

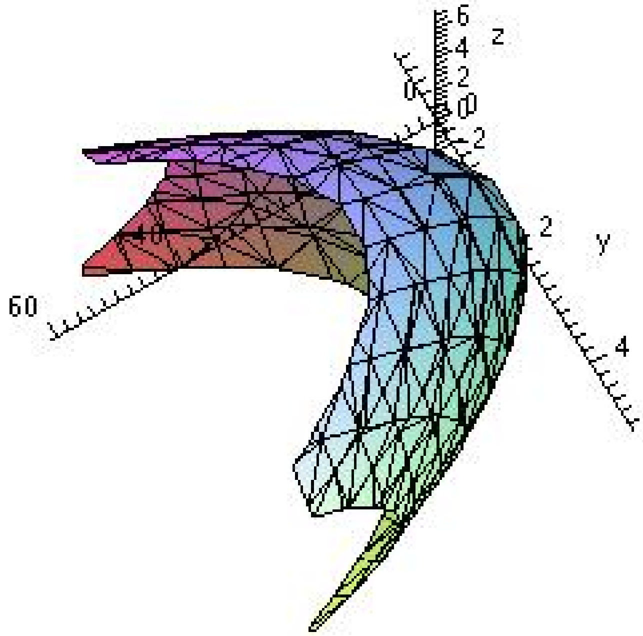

Figure 2: Plot for #34

$2 - 6x - 5y = 0$, which is a plane with normal direction as $< 6, 5, 0 >$. The curve is included in this plane.

13.7: #46

Note we have $\rho^2 = x^2 + y^2$, so the equation turned out to be $x^2 + y^2 - 4z^2 = 1$, which is a hyperboloid of one sheet with z-axis as the axis of symmetry.

13.7: #50

Cylindrical: $r^2 + z^2 = 2$
Spherical: $\rho^2 = 2$

13.7: #58

It is a quarter cone. See figure 5.

13.7: #68

The great circle distance is the distance along the surface of the sphere (Earth) that one would have to travel between the two points. The first step is to translate the points into normal spherical coordinates.

$$\phi_1 = 90^\circ - 34.06^\circ = 55.94^\circ$$
$$\theta_1 = 360^\circ - 118.25^\circ = 241.75^\circ$$
$$\phi_2 = 90^\circ - 45.50^\circ = 44.50^\circ$$
$$\theta_2 = 360^\circ - 73.60^\circ = 286.4^\circ$$

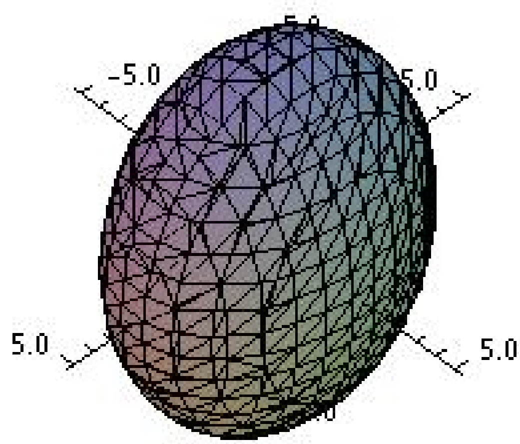

Figure 3: Plot for #38.

Now, using $\rho = 3960$ mi., convert to rectangular coordinates. This will allow us to get the angle between the points on the great circle by calculating a dot product.

$$\begin{aligned}
x_1 = \rho * \sin(\phi_1) * \cos(\theta_1) &= -1552.8 \\
y_1 = \rho * \sin(\phi_1) * \sin(\theta_1) &= -2889.91 \\
z_1 = \rho * \cos(\phi_1) &= 2217.84 \\
x_2 = \rho * \sin(\phi_2) * \cos(\theta_2) &= 783.67 \\
y_2 = \rho * \sin(\phi_2) * \sin(\theta_2) &= -2662.67 \\
z_2 = \rho * \cos(\phi_2) &= 2824.47
\end{aligned}$$

Now consider two vectors, one from the center of the Earth to Montreal and one from the center to LA. The angle between these vectors will be the angle of the sector of the great circle between these points. So we use:

$$\theta = \cos^{-1}\left(\frac{\mathbf{v_1} \cdot \mathbf{v_2}}{|\mathbf{v_1}||\mathbf{v_2}|}\right)$$

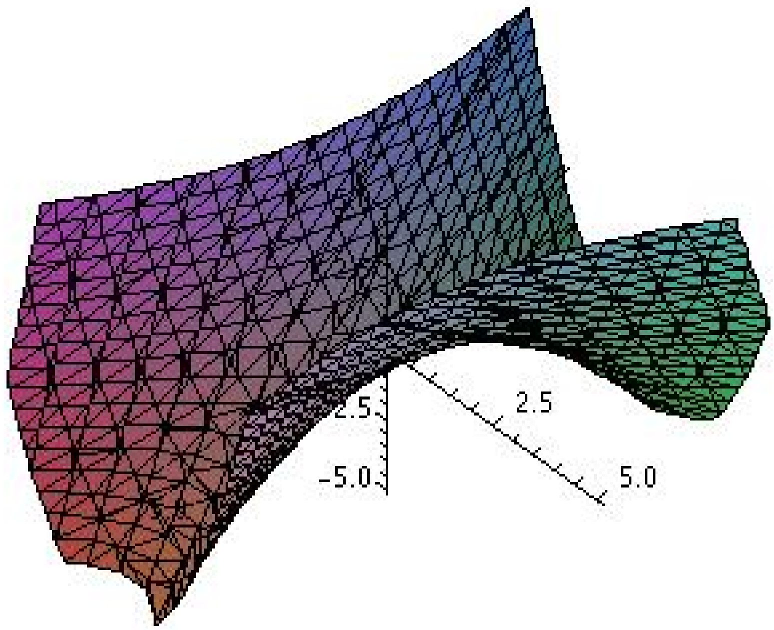

Figure 4: Plot for #40.

$$\begin{aligned} \theta &= \cos^{-1}\left(\tfrac{1.274\cdot 10^7}{1.568\cdot 10^7}\right) \\ \theta &= 0.6223 \end{aligned}$$

So the distance is the arc length of the great circle, $d = \rho \cdot \theta = 3960$ mi $\cdot 0.6223 = 2464.175$ mi. A Mapquest check for the driving distance between the two cities gives 2850 miles, which is a little larger than the great circle distance, just as we'd expect.

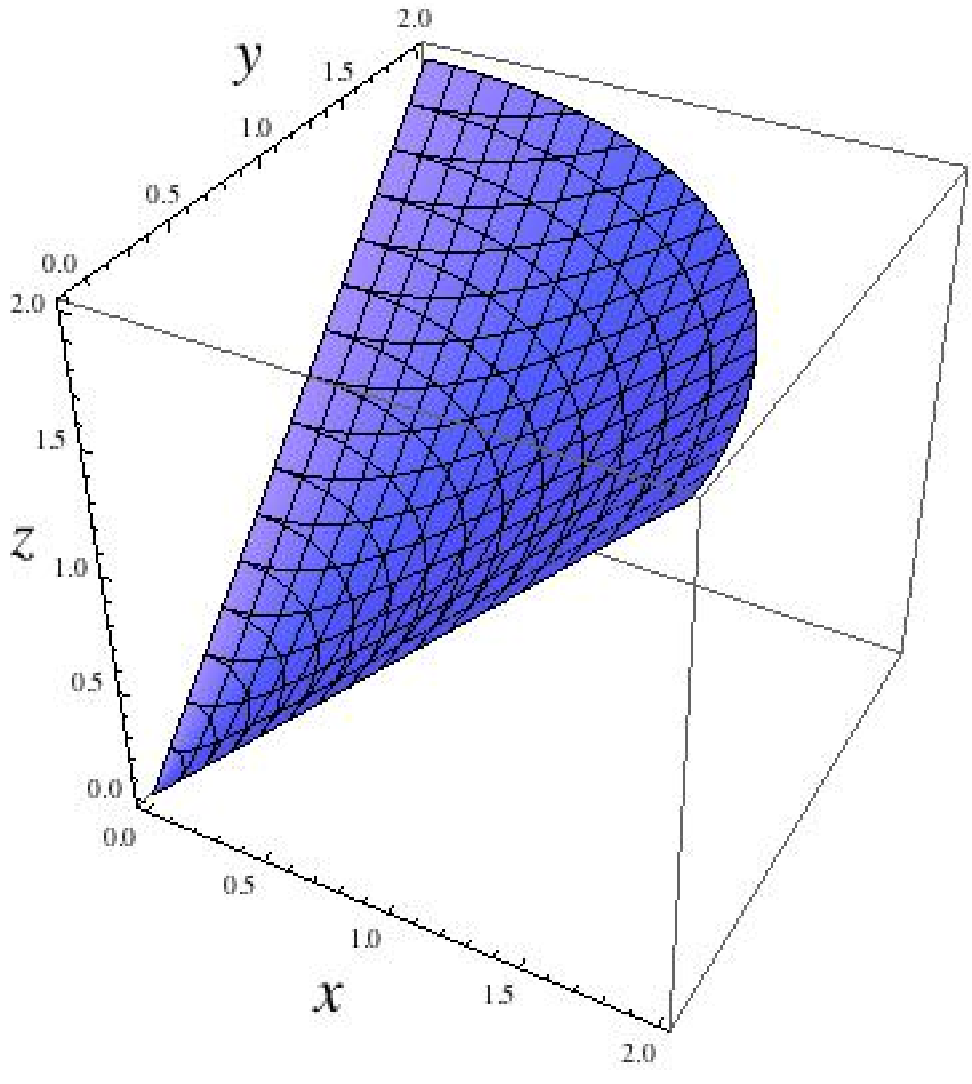

Figure 5: Plot for #58