MATH 255 **Applied Honors Calculus III** **Winter 2011**

Homework 7

Due: Tuesday, March 15, 2011

Note: In what follows, numbers in parentheses indicate the problem numbers for users of the sixth edition. A * indicates that this problem is not in the sixth edition and that you should look in the Michigan edition to find it.

Section 15.8, pg. 1007: 10, 30 (*), 40 (42), 43 (45).
Section 16.1, pg. 1025: 10(*), 12, 18 (*).
Section 16.2, pg. 1030: 10 (*), 12 (*), 14 (16), 16 (18), 36 (38).
Section 16.3, pg. 1038: 4 (*), 18, 32, 50 (*), 54 (58).

Additional Problem:

An Interpretation of λ
Consider the problem of maximizing a function $f(x,y)$ subject to the constraint $g(x,y)=c$. We want to investigate the how the maximum value changes as c does.
When the constraint is $g(x,y)=c$, suppose the maximum value is at the point $(x^*(c),y^*(c))$ and that the corresponding Lagrange multiplier is $\lambda^*(c)$. We are going to assume that all of these functions are differentiable (as functions of c) - this is not unreasonable if $f(x,y)$ and $g(x,y)$ are sufficiently differentiable. Further assume that $g_x(x^*(c),y^*(c)) \neq 0$ and $g_y(x^*(c),y^*(c)) \neq 0$.
Set $f^*(c) = f(x^*(c),y^*(c))$, the maximum value of the function subject to the constraint being equal to c.
a) Show that

$$f'^*(c) = \lambda^*(c)[g_x(x^*(c),y^*(c))x'^*(c) + g_y(x^*(c),y^*(c))y'^*(c)].$$

b) And conclude that

$$f'^*(c) = \lambda^*(c).$$

c) Write a complete sentence explaining the meaning of the value of λ in the method of Lagrange multipliers.